Regular Article

Performance Analysis of Adaptive Modulation for Distributed Switch-and-Stay Combining with Single Relay

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Abstract– **In this paper we present our investigation of the performance of a distributed switch-and-stay scheme and a solution for low-complexity cooperative relaying under adaptive transmission. We are able to derive the closed-form expressions of the occurrence probability, the outage probability, the bit error probability, and the achievable spectral efficiency of the proposed scheme for Rayleigh fading channels. Simulations are performed to verify the analytic results. It is shown that, by applying adaptive modulation, the achievable spectral efficiency of the system is improved significantly, and the proposed scheme achieves the same spectral efficiency as compared to that of the incremental relaying but with a lower relay activation time.**

Keywords– **Switch-and-stay combining, amplify-and-forward, Rayleigh fading, outage probability, spectral efficiency, bit error rate, adaptive transmission, incremental relaying.**

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1 Introduction

For communication over wireless fading channels, cooperative systems have been shown to offer a tremendous spatial diversity gain in capacity and in network coverage when compared to conventional direct transmission, since installing multiple antennas on wireless agents is impossible [1]. However, this gain is obtained at the cost of lower spectral efficiency and higher complexity because at least two time-slot periods are needed to transmit one data packet, and combining techniques, such as maximal ratio combining (MRC) and selection combining (SC), are expected at the destination [2].

To overcome such problems while still maintaining the spatial diversity, the concept of switched diversity is applied for cooperative networks, promising a significant reduction in system complexity as well as an improvement in system spectral efficiency [3–7]. In particular, the concept of distributed switch-and-stay (DSSC) was first introduced to single-relay cooperative networks in [3] and [5]. It was then extended to two-relay networks in [4]. Recently, to offer more spatial diversity, DSSC networks in conjunction with relay selection were proposed and studied in [6, 7].

In this paper, we propose to apply adaptive modulation to DSSC networks with a single relay to further improve the spectral efficiency. The same approach can be proposed for networks with multiple relays. The fundamental idea is that, based on a limited feedback signal, a source can adapt its transmitted constellation size according to channel conditions to efficiently achieve a higher data rate for a given target bit error rate, BER_T.

There have been various works on the performance analysis of cooperative systems in conjunction with adaptive modulation [8–11]. In particular, the performance analysis of repetition-based amplify-and-forward (AF) cooperative networks with five-mode M-QAM over Rayleigh fading channels was presented in [8]. In [9], Hwang *et al.* proposed an incremental relaying protocol based on an AF transmission in conjunction with the best relay selection scheme over non-identically distributed relaying channels. Their experimental results showed that this scheme could provide a certain improvement of the spectral efficiency and the outage probability while achieving the desired BER.

In addition to AF networks, adaptive modulation was also proposed for repetition-based decode-and-forward networks in [10] where both optimal and fixed switching were investigated. More recently, the work in [11] was devoted to analyze the performance of adaptive discrete modulation for opportunistic AF relaying networks. Based on the probability density function (PDF) of the end-to-end signal-to-noise (SNR), which was expressed under a mathematically tractable form, important metrics of these networks, including the outage probability, the spectral efficiency and the bit error rate, were derived.

In this paper we present our study of the performance of DSSC networks with a single relay, using adaptive modulation. The importance of our contribution is twofold. Firstly, we derived several important performance metrics of the systems: the occurrence probability, the outage probability, the spectral efficiency and the bit error probability, for Rayleigh fading channels. Secondly, for a given target BER, we obtained the optimal boundary thresholds which maximize the system spectral efficiency.

Additionally, we studied the performance of incremental relaying (IR) networks as a counterpart of the DSSC networks. The similar characteristic of both systems is that they attain the beneficial effects of diversity without employing any diversity combiner at the destination by utilizing appropriate feedbacks sent by the destination. The main difference between them is the switching direction, i.e., an IR network is considered as a special case of the DSSC networks, but switching in the IR network is limited to only one direction on the direct link and the relaying link, while both directions are applicable in the DSSC networks.

The paper is organized as follows. In Section 2, we introduce the system model under study and describe the proposed protocol. Section 3 presents the derivation of the outage probability and the achievable spectral efficiency for the proposed system. Section 4 provides simulation results, comparisons between simulated and theoretical results, and discussions on the behavior of the proposed system at low and high SNR regimes. Finally, a summary of the results of this work is presented in Section 5.

2 System Model

We use the same DSSC network model with a single relay as in [3]. Assume that the network under consideration, as shown in Figure 1, consists of a single source ($\mathcal{S}$), an AF relay ($\mathcal{R}$) and a single destination ($\mathcal{D}$), and operates over Rayleigh fading channels. Assume also that all nodes operate in the half-duplex mode and the source signal reaches to the destination by either the direct link ($\mathcal{S} \to \mathcal{D}$) or the relaying link ($\mathcal{S} \to \mathcal{R} \to \mathcal{D}$), according to the rules of SSC combining [12, 13]. Only one link is active in each transmission slot. At the destination, the active link is driven by comparing the received instantaneous SNR with a given threshold T. Switching will not occur as long as the instantaneous SNR on the currently active link remains above T. Under the slow-fading assumption, the fading envelope is assumed to be constant over at least two consecutive transmission slots. Therefore, the switching process becomes effective during the following transmission slot with the help of a limited feedback sent to the source and the relay.

When the relaying link is active, the transmission slot is divided into two sub-slots. During the first sub-slot, the source transmits its signal s with an average transmit power $\mathcal{P}_s$ to the relay. In the second sub-slot,

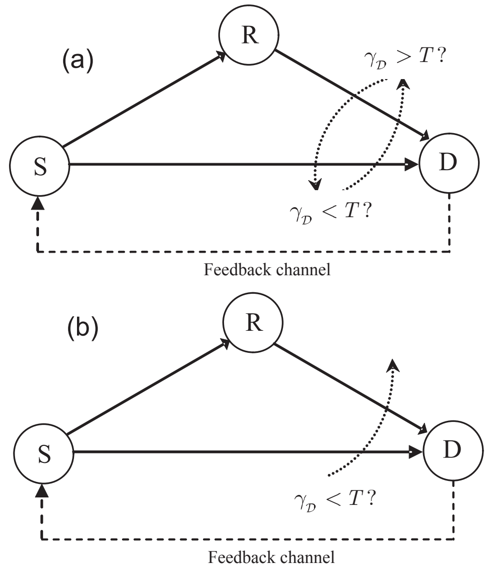

Figure 1. (a) The DSSC network. (b) The IF network.

the relay amplifies and, then, forwards the received signals towards the destination with an average transmit power $\mathcal{P}_r$. Let $h_{\mathcal{AB}}$ be the link coefficient between a transmitting node $\mathcal{A}$ and a reception node $\mathcal{B}$. The signals received at the relay and the destination are, respectively, mathematically modeled by the following set of equations:

$$r_{\mathcal{SR}} = \sqrt{\mathcal{P}_s} h_{\mathcal{SR}} s + n_{\mathcal{SR}}, \tag{1}$$

$$r_{\mathcal{RD}} = \sqrt{\mathcal{P}_r} h_{\mathcal{RD}} \mathcal{G} r_{\mathcal{SR}} + n_{\mathcal{RD}}, \tag{2}$$

where $n_{\mathcal{SR}}$ and $n_{\mathcal{RD}}$ are the additive white Gaussian noise (AWGN) samples (having zero mean and variance $\mathcal{N}_0$) at $\mathcal{R}$ and $\mathcal{D}$, respectively. The amplifying factor, $\mathcal{G}$, is defined as

$$\mathcal{G} = \sqrt{\frac{\mathcal{P}_r}{\mathcal{P}_s |h_{\mathcal{SR}}|^2 + \mathcal{N}_0}}. \tag{3}$$

When the direct link is active, the received signal at the destination is given by

$$r_{\mathcal{SD}} = \mathcal{P}_s h_{\mathcal{SD}} s + n_{\mathcal{SD}}. \tag{4}$$

Let $\gamma_{\mathcal{D}}$, $\gamma_{\mathcal{SR}}$ and $\gamma_{\mathcal{RD}}$ be the instantaneous SNRs for $\mathcal{S} \to \mathcal{D}$, $\mathcal{S} \to \mathcal{R}$, and $\mathcal{R} \to \mathcal{D}$ links, respectively; $\gamma_{\mathcal{D}} = \mathcal{P}_s |h_{\mathcal{SD}}|^2 / \mathcal{N}_0$, $\gamma_{\mathcal{SR}} = \mathcal{P}_s |h_{\mathcal{SR}}|^2 / \mathcal{N}_0$, and $\gamma_{\mathcal{RD}} = \mathcal{P}_r |h_{\mathcal{RD}}|^2 / \mathcal{N}_0$. From (1) and (2), the equivalent instantaneous SNR of the $\mathcal{S} \to \mathcal{R} \to \mathcal{D}$ link is given by [1]

$$\gamma_{\mathcal{R}} = \frac{\gamma_{\mathcal{SR}} \gamma_{\mathcal{RD}}}{\gamma_{\mathcal{SR}} + \gamma_{\mathcal{RD}} + 1}. \tag{5}$$

Making use of the fact that in medium and high SNR regimes dual-hop AF relaying is dominated by the weakest hop, (5) can be well-approximated by [14, 15]

$$\gamma_{\mathcal{R}} \approx \min\{\gamma_{\mathcal{SR}}, \gamma_{\mathcal{RD}}\}. \tag{6}$$

Since $\gamma_{\mathcal{SR}}$ and $\gamma_{\mathcal{RD}}$ are exponentially distributed, $\gamma_{\mathcal{R}}$ is also an exponential random variable [16]. The PDF and the cumulative probability function (CDF) of $\gamma_{\mathcal{R}}$ are therefore, respectively, expressed as follows:

$$f_{\gamma_{\mathcal{R}}}(\gamma) = \frac{1}{\bar{\gamma}_{\mathcal{R}}} e^{-\frac{\gamma}{\bar{\gamma}_{\mathcal{R}}}}, \quad (7)$$

$$F_{\gamma_{\mathcal{R}}}(\gamma) = \int_0^{\gamma} f_{\gamma_{\mathcal{R}}}(\gamma) d\gamma = 1 - e^{-\frac{\gamma}{\bar{\gamma}_{\mathcal{R}}}}, \quad (8)$$

where $\bar{\gamma}_{\mathcal{R}} = (\bar{\gamma}_{\mathcal{SR}}\bar{\gamma}_{\mathcal{RD}})/(\bar{\gamma}_{\mathcal{SR}} + \bar{\gamma}_{\mathcal{RD}})$ [16, Eq. (6.82)], with $\bar{\gamma}_{\mathcal{SR}} = E\{\gamma_{\mathcal{SR}}\}$ and $\bar{\gamma}_{\mathcal{RD}} = E\{\gamma_{\mathcal{RD}}\}$; here, $E\{.\}$ denotes the expectation.

For the direct link, under a flat Rayleigh fading channel, the PDF and the CDF of $\gamma_{\mathcal{D}}$ can be expressed as

$$f_{\gamma_{\mathcal{D}}}(\gamma) = \frac{1}{\bar{\gamma}_{\mathcal{D}}} e^{-\frac{\gamma}{\bar{\gamma}_{\mathcal{D}}}}, \quad (9)$$

$$F_{\gamma_{\mathcal{D}}}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_{\mathcal{D}}}}, \quad (10)$$

where $\bar{\gamma}_{\mathcal{D}} = E\{\gamma_{\mathcal{D}}\}$.

According to the operation mode of adaptive discrete modulation, we partition the entire effective received SNR range into K non-overlapping intervals, determined by a set of boundary values denoted as $0 = \gamma_T^0 < \gamma_T^1 < \cdots < \gamma_T^k < \cdots < \gamma_T^K = +\infty$. Since the end-to-end instantaneous received SNR, γ_Σ, is in the interval $[\gamma_T^{k-1}, \gamma_T^k)$, the destination will choose a modulation scheme M_k-QAM among the K possible schemes to ensure that the bit error probability of each scheme is below the target bit error rate, BER_T. After that, the destination sends this information back to the source through a feedback channel. To facilitate the analysis, we assume that the feedback channel is error-free with no delay. Furthermore, to avoid deep fadings, there will be no data sent when the end-to-end instantaneous received SNR is lower than γ_T^1.

To determine the fixed boundary values, we start with the bit error rate of M-QAM with Gray coding over additive white Gaussian noise channels [17, Table 6.1], namely

$$P_{\mathrm{b}}^{\mathrm{QAM}}(k, \gamma_\Sigma) \approx \alpha_k Q\left(\sqrt{\beta_k \gamma_\Sigma}\right), \quad (11)$$

where, with $m_k \triangleq \log_2 M_k$,

$$\alpha_k = \begin{cases} 1, & m_k = 1,2 \\ 4/m_k, & m_k \geq 3 \end{cases}$$

$$\beta_k = \begin{cases} 2/m_k, & m_k = 1,2 \\ 3/(2^{m_k} - 1), & m_k \geq 3 \end{cases}$$

The boundary value relative to each mode can be computed, by inverting (11) and assigning the SNR to satisfy the target BER, as follows:

$$\gamma_T^k = \frac{1}{\beta_k}\left[Q^{-1}\left(\frac{\mathrm{BER}_T}{\alpha_k}\right)\right]^2, \quad k = 1 \ldots K-1. \quad (12)$$

3 Performance Analysis

3.1 DSSC Networks

Before investigating the performance of DSSC networks under adaptive modulation, we first study the percentage of time (or, equivalently, the connection probability) during which the direct and the relaying links are connected to the destination. Recall that switching will occur if the SNR of the currently connected link falls below the switching threshold, T. According to [13, eq. (9.326)], we have the following:

$$p_{\mathcal{D}} = \frac{\Pr(\gamma_{\mathcal{R}} < T)}{\Pr(\gamma_{\mathcal{D}} < T) + \Pr(\gamma_{\mathcal{R}} < T)} = \frac{F_{\gamma_{\mathcal{R}}}(T)}{F_{\gamma_{\mathcal{D}}}(T) + F_{\gamma_{\mathcal{R}}}(T)}, \quad (13)$$

$$p_{\mathcal{R}} = \frac{\Pr(\gamma_{\mathcal{D}} < T)}{\Pr(\gamma_{\mathcal{D}} < T) + \Pr(\gamma_{\mathcal{R}} < T)} = \frac{F_{\gamma_{\mathcal{D}}}(T)}{F_{\gamma_{\mathcal{D}}}(T) + F_{\gamma_{\mathcal{R}}}(T)}. \quad (14)$$

Above, $F_{\gamma_Z}(T) = \Pr(\gamma_Z < T) = 1 - \Pr(\gamma_Z \geq T)$, with $Z \in \{\mathcal{D}, \mathcal{R}\}$, can be trivially obtained by evaluating $F_{\gamma_Z}(\gamma)$ at $\gamma = T$, respectively.

3.1.1 Occurrence probability of each mode: The occurrence probability of mode k ($k = 1, \ldots, K$) can generally be expressed as

$$\begin{aligned}\pi_k = & p_{\mathcal{D}} \times \Big[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k) + \\ & \Pr(\gamma_{\mathcal{D}} > T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k | (\gamma_{\mathcal{D}} > T))\Big] + \\ & p_{\mathcal{R}} \times \Big[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k) + \\ & \Pr(\gamma_{\mathcal{R}} > T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k | (\gamma_{\mathcal{R}} > T))\Big]. \end{aligned} \quad (15)$$

For a given arbitrary value of T and taking into account all possible cases of T relative to[1] $[\gamma_T^{k-1}, \gamma_T^k)$, [18], π_k is re-written as

$$\pi_k = \begin{cases} \pi_k^{(1)}, & \gamma_T^k < T \\ \pi_k^{(2)}, & \gamma_T^{k-1} < T \leq \gamma_T^k \\ \pi_k^{(3)}, & T < \gamma_T^{k-1} \end{cases} \quad (16)$$

where

$$\begin{aligned}\pi_k^{(1)} = & p_{\mathcal{D}}\Big[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k)\Big] + \\ & p_{\mathcal{R}}\Big[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k)\Big], \\ \pi_k^{(2)} = & p_{\mathcal{D}}\Big[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k) + \\ & \Pr(T < \gamma_{\mathcal{D}} \leq \gamma_T^k)\Big] + \\ & p_{\mathcal{R}}\Big[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k) + \\ & \Pr(T < \gamma_{\mathcal{R}} \leq \gamma_T^k)\Big], \\ \pi_k^{(3)} = & p_{\mathcal{D}}\Big[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k) + \\ & \Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k)\Big] + \\ & p_{\mathcal{R}}\Big[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k) + \\ & \Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k)\Big]. \end{aligned}$$

In the above, $\Pr(x < \gamma_Z \leq \gamma_T^k) = F_{\gamma_Z}(\gamma_T^k) - F_{\gamma_Z}(x)$, $x \in \{T, \gamma_T^{k-1}\}$.

3.1.2 Outage probability: To avoid deep fadings, DSSC systems with adaptive modulation stop data transmission when the end-to-end SNR is in $[\gamma_T^0, \gamma_T^1)$, thereby suffering a probability of outage. In other words, the outage probability of these systems, OP, defined as the probability that the end-to-end SNR falls below γ_T^1, is equal to π_1, namely

$$\mathrm{OP} = \pi_1.$$

[1]It is noted that the case of $T < \gamma_T^{k-1}$, i.e., $\pi_1^{(3)}$, does not exist for $k = 1$.

Recall the fact that T could be smaller than, equal to or greater than γ_T^1, which leads to two cases that need to be considered separately. To this end, the system outage probability is given by

$$\mathrm{OP} = \begin{cases} \pi_1^{(1)}, & \gamma_T^1 < T \\ \pi_1^{(2)}, & \gamma_T^0 < T \leq \gamma_T^1 \end{cases} \tag{17}$$

where

$$\begin{aligned}\pi_1^{(1)} =& p_{\mathcal{D}} \left[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^0 < \gamma_{\mathcal{R}} \leq \gamma_T^1)\right] + \\ & p_{\mathcal{R}} \left[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^0 < \gamma_{\mathcal{D}} \leq \gamma_T^1)\right], \\ \pi_1^{(2)} =& p_{\mathcal{D}} \left[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^0 < \gamma_{\mathcal{R}} \leq \gamma_T^1) + \Pr(T < \gamma_{\mathcal{D}} \leq \gamma_T^1)\right] + \\ & p_{\mathcal{R}} \left[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^0 < \gamma_{\mathcal{D}} \leq \gamma_T^1) + \Pr(T < \gamma_{\mathcal{R}} \leq \gamma_T^1)\right].\end{aligned}$$

3.1.3 Average spectral efficiency: The average achievable spectral efficiency is simply the sum of the data rates in all partitioned regions, weighted by their corresponding probabilities of occurrence. Making use of the fact that the spectral efficiency for $\gamma_\Sigma \in [\gamma_T^{k-1}, \gamma_T^k)$, with $k > 1$, is $\log_2(M_k)$ bps/Hz for the direct link, and $\frac{1}{2}\log_2(M_k)$ bps/Hz for the relaying link, we have

$$\mathrm{ASE} = \sum_{k=1}^{K-1} \eta_k. \tag{18}$$

In (18), we have

$$\eta_k = \begin{cases} \eta_k^{(1)}, & \gamma_T^{k+1} < T \\ \eta_k^{(2)}, & \gamma_T^k < T \leq \gamma_T^{k+1} \\ \eta_k^{(3)}, & T < \gamma_T^k \end{cases}$$

where

$$\begin{aligned}\eta_k^{(1)} =& p_{\mathcal{D}} \left[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k)m_k/2\right] + \\ & p_{\mathcal{R}} \left[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k)m_k\right], \\ \eta_k^{(2)} =& p_{\mathcal{D}} \left[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k)m_k/2 + \Pr(T < \gamma_{\mathcal{D}} \leq \gamma_T^k)m_k\right] + \\ & p_{\mathcal{R}} \left[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k)m_k + \Pr(T < \gamma_{\mathcal{R}} \leq \gamma_T^k)m_k\right], \\ \eta_k^{(3)} =& p_{\mathcal{D}} \left[\Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k)m_k/2 + \Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k)m_k\right] + \\ & p_{\mathcal{R}} \left[\Pr(\gamma_{\mathcal{R}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k)m_k + \Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k)m_k\right].\end{aligned}$$

3.1.4 Bit error rate: The average BER for DSSC networks with adaptive modulation can be calculated by [19]

$$\overline{\mathrm{BER}} = \frac{\sum_{k=2}^{K} m_k \overline{\mathrm{BER}}_k}{\sum_{k=2}^{K} m_k \pi_k}. \tag{19}$$

In (19), $\overline{\mathrm{BER}}_k$ denotes the average BER in a specific region of $[\gamma_T^k, \gamma_T^{k+1})$, and is given by

$$\overline{\mathrm{BER}}_k = \begin{cases} \overline{\mathrm{BER}}_k^{(1)}, & \gamma_T^k < T \\ \overline{\mathrm{BER}}_k^{(2)}, & \gamma_T^{k-1} < T \leq \gamma_T^k \\ \overline{\mathrm{BER}}_k^{(3)}, & T < \gamma_T^{k-1} \end{cases} \tag{20}$$

where

$$\begin{aligned}\overline{\mathrm{BER}}_k^{(1)} =& p_{\mathcal{D}} \Pr(\gamma_{\mathcal{D}} \leq T)\mathcal{I}_{\mathcal{R}}^k(\gamma_T^{k-1}, \gamma_T^k) + \\ & p_{\mathcal{R}} \Pr(\gamma_{\mathcal{R}} \leq T)\mathcal{I}_{\mathcal{D}}^k(\gamma_T^{k-1}, \gamma_T^k), \\ \overline{\mathrm{BER}}_k^{(2)} =& p_{\mathcal{D}} \left[\Pr(\gamma_{\mathcal{D}} \leq T)\mathcal{I}_{\mathcal{R}}^k(\gamma_T^{k-1}, \gamma_T^k) + \mathcal{I}_{\mathcal{D}}^k(T, \gamma_T^k)\right] + \\ & p_{\mathcal{R}} \left[\Pr(\gamma_{\mathcal{R}} \leq T)\mathcal{I}_{\mathcal{D}}^k(\gamma_T^{k-1}, \gamma_T^k) + \mathcal{I}_{\mathcal{R}}^k(T, \gamma_T^k)\right], \\ \overline{\mathrm{BER}}_k^{(3)} =& p_{\mathcal{D}} \left[\Pr(\gamma_{\mathcal{D}} \leq T)\mathcal{I}_{\mathcal{R}}^k(\gamma_T^{k-1}, \gamma_T^k) + \mathcal{I}_{\mathcal{D}}^k(\gamma_T^{k-1}, \gamma_T^k)\right] + \\ & p_{\mathcal{R}} \left[\Pr(\gamma_{\mathcal{R}} \leq T)\mathcal{I}_{\mathcal{D}}^k(\gamma_T^{k-1}, \gamma_T^k) + \mathcal{I}_{\mathcal{R}}^k(\gamma_T^{k-1}, \gamma_T^k)\right].\end{aligned}$$

Furthermore, $\mathcal{I}_Z^k(x, \gamma_T^k)$ is defined as follows:

$$\mathcal{I}_Z^k(x, \gamma_T^k) = \int_x^{\gamma_T^k} P_{\mathrm{b}}^{\mathrm{QAM}}(k, \gamma) f_Z(\gamma) d\gamma. \tag{21}$$

Substituting $P_{\mathrm{b}}^{\mathrm{QAM}}(k, \gamma)$ in (21) by (11) yields a closed-form solution for $\mathcal{I}_Z^k(x, \gamma_T^k)$, as shown in (22) at the top of the next page.

3.2 IF Networks

In IR networks with adaptive modulation, the destination estimates its received SNR and determines the possible constellation size based on the direct signal sent by the source if the SNR of the direct link is greater than a pre-determined threshold, i.e., $\gamma_{\mathcal{D}} > T$. Otherwise, the destination requests help from the relay. It needs to be emphasized that in the latter case the destination does not utilize a diversity combiner, e.g., MRC or SC. Consequently, the final constellation size is decided based on the instantaneous SNR of the dual-hop relaying link and this information is sent to the source via the feedback link.

3.2.1 Occurrence probability: Worked similarly, we obtain the occurrence probability of mode k given by

$$\begin{aligned}\pi_k =& \Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k) + \\ & \Pr(\gamma_{\mathcal{D}} > T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k | (\gamma_{\mathcal{D}} > T)) \\ =& \begin{cases} \pi_k^{(1)}, & \gamma_T^k < T \\ \pi_k^{(2)}, & \gamma_T^{k-1} < T \leq \gamma_T^k \\ \pi_k^{(3)}, & T < \gamma_T^{k-1} \end{cases}\end{aligned} \tag{23}$$

where

$$\begin{aligned}\pi_k^{(1)} =& \Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k), \\ \pi_k^{(2)} =& \Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k) + \\ & \Pr(T < \gamma_{\mathcal{D}} \leq \gamma_T^k), \\ \pi_k^{(3)} =& \Pr(\gamma_{\mathcal{D}} \leq T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \leq \gamma_T^k) + \\ & \Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \leq \gamma_T^k).\end{aligned}$$

$$\mathcal{I}_Z^k(x,\gamma_T^k) = \int_x^{\gamma_T^k} \alpha_k Q\left(\sqrt{\beta_k\gamma}\right)\frac{1}{\bar\gamma_Z}e^{-\frac{\gamma}{\bar\gamma_Z}}d\gamma$$
$$= \alpha_k\left\{Q\left(\sqrt{\beta_k\gamma}\right)\left(1-e^{-\frac{\gamma}{\bar\gamma_Z}}\right)-\frac{1}{2}\left[\sqrt{\frac{1}{\pi}}\Gamma\left(\frac{1}{2},\frac{\beta_k\gamma}{2}\right)-\sqrt{\frac{\beta_k}{2\pi}}\left(\frac{\beta_k}{2}+\frac{1}{\bar\gamma_Z}\right)^{-\frac{1}{2}}\Gamma\left(\frac{1}{2},\left(\frac{\beta_k}{2}+\frac{1}{\bar\gamma_Z}\right)\gamma\right)\right]\right\}_x^{\gamma_T^k} \quad (22)$$

3.2.2 Outage probability: It is straightforwardly recognized that the outage probability of the IR scheme is, again, equal to the occurrence probability of the first mode, namely

$$\mathrm{OP} = \pi_1 = \begin{cases}\pi_1^{(1)}, & \gamma_T^1 < T\\ \pi_1^{(2)}, & \gamma_T^0 < T \le \gamma_T^1\end{cases} \quad (24)$$

where

$$\pi_1^{(1)} = \Pr(\gamma_{\mathcal{D}} \le T)\Pr(\gamma_T^0 < \gamma_{\mathcal{R}} \le \gamma_T^1),$$
$$\pi_1^{(2)} = \Pr(\gamma_{\mathcal{D}} \le T)\Pr(\gamma_T^0 < \gamma_{\mathcal{R}} \le \gamma_T^1) + \Pr(T < \gamma_{\mathcal{D}} \le \gamma_T^1).$$

3.2.3 Average spectral efficiency: Under IR, the relay node is used only when the direct link undergoes an outage, i.e., when $\gamma_{\mathcal{D}} < T$, and it can achieve a spectral efficiency of only $\frac{1}{2}m_k$ bps/Hz. Using the same approach for the occurrence probability, the average spectral efficiency, defined as the spectral efficiency in a long-term perspective, is given by

$$\mathrm{ASE} = \sum_{k=2}^{K}\eta_k. \quad (25)$$

In (25),

$$\eta_k = \begin{cases}\eta_k^{(1)}, & \gamma_T^k < T\\ \eta_k^{(2)}, & \gamma_T^{k-1} < T \le \gamma_T^k\ ,\\ \eta_k^{(3)}, & T < \gamma_T^{k-1},\end{cases}$$

where

$$\eta_k^{(1)} = \Pr(\gamma_{\mathcal{D}} \le T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \le \gamma_T^k)m_k/2,$$
$$\eta_k^{(2)} = \Pr(\gamma_{\mathcal{D}} \le T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \le \gamma_T^k)m_k/2 + \Pr(T < \gamma_{\mathcal{D}} \le \gamma_T^k)m_k,$$
$$\eta_k^{(3)} = \Pr(\gamma_{\mathcal{D}} \le T)\Pr(\gamma_T^{k-1} < \gamma_{\mathcal{R}} \le \gamma_T^k)m_k/2 + \Pr(\gamma_T^{k-1} < \gamma_{\mathcal{D}} \le \gamma_T^k)m_k.$$

3.2.4 Bit error rate: Likewise, the average BER for IR networks with adaptive modulation is given by

$$\overline{\mathrm{BER}} = \frac{\sum_{k=2}^{K} m_k\overline{\mathrm{BER}}_k}{\sum_{k=2}^{K} m_k\pi_k}. \quad (26)$$

In (26),

$$\overline{\mathrm{BER}}_k = \begin{cases}\overline{\mathrm{BER}}_k^{(1)}, & \gamma_T^k < T\\ \overline{\mathrm{BER}}_k^{(2)}, & \gamma_T^{k-1} < T \le \gamma_T^k\\ \overline{\mathrm{BER}}_k^{(3)}, & T < \gamma_T^{k-1}\end{cases}$$

where

$$\overline{\mathrm{BER}}_k^{(1)} = \Pr(\gamma_{\mathcal{D}} \le T)\mathcal{I}_{\mathcal{R}}^k(\gamma_T^{k-1},\gamma_T^k),$$
$$\overline{\mathrm{BER}}_k^{(2)} = \Pr(\gamma_{\mathcal{D}} \le T)\mathcal{I}_{\mathcal{R}}^k(\gamma_T^{k-1},\gamma_T^k) + \mathcal{I}_{\mathcal{D}}^k(T,\gamma_T^k),$$
$$\overline{\mathrm{BER}}_k^{(3)} = \Pr(\gamma_{\mathcal{D}} \le T)\mathcal{I}_{\mathcal{R}}^k(\gamma_T^{k-1},\gamma_T^k) + \mathcal{I}_{\mathcal{D}}^k(\gamma_T^{k-1},\gamma_T^k).$$

3.3 Maximizing ASE

For both schemes, by using discrete adaptive modulation, the source is able to communicate with the destination at a bit rate as high as possible while ensuring the target BER. Therefore, the boundary values play an important role in system performance. In a fixed mode, these values are directly calculated based on the instantaneous BER by (12). However, that is not an optimal way since the actual average BER is always below the target BER, implying that the average spectral efficiency of the system could potentially be further improved. For a given T, the optimization problem reads

$$\begin{aligned}&\underset{\gamma_T^k, k=1,\ldots,K}{\text{maximize}} && \mathrm{ASE},\\ &\text{subject to} && \overline{\mathrm{BER}} \le \mathrm{BER}_T\,.\end{aligned} \quad (27)$$

With the ASE obtained in closed forms as in (18) and (25), we can solve the above optimization problem by using the Lagrangian technique [20]. Specifically, following the approach in [21], the set of $K-1$ optimized switching thresholds for a particular average transmit power and a target BER can be found by solving the following $K-1$ nonlinear equations:

$$\begin{cases}\sum_{k=1}^{K-1} m_k\overline{\mathrm{BER}}_k - \mathrm{BER}_T\sum_{k=1}^{K-1} m_k\pi_k = 0,\\ y_1(\gamma_T^1) - y_k(\gamma_T^k) = 0, \quad k = 2,\ldots,K,\end{cases} \quad (28)$$

where

$$y_m(\gamma_T^k) = \frac{m_k P_b^{\mathrm{QAM}}(m_k,\gamma_T^k) - m_{k-1}P_b^{\mathrm{QAM}}(m_{k-1},\gamma_T^{k-1})}{m_k - m_{k-1}}.$$

4 Numerical Results and Discussions

We performed MATLAB-based computer simulations to verify the theoretical analysis. In all simulations, the variances of the channel fadings of the three links are set as $E\{|h_{\mathcal{SD}}|^2\} = 1$, $E\{|h_{\mathcal{SR}}|^2\} = 2$ and $E\{|h_{\mathcal{RD}}|^2\} = 3$, respectively, and the five-mode adaptive M-QAM is employed as in [8, 10, 11]. Furthermore, the transmit power of the source is assumed to be equal to that of the relay, i.e., $\mathcal{P}_s = \mathcal{P}_r$.

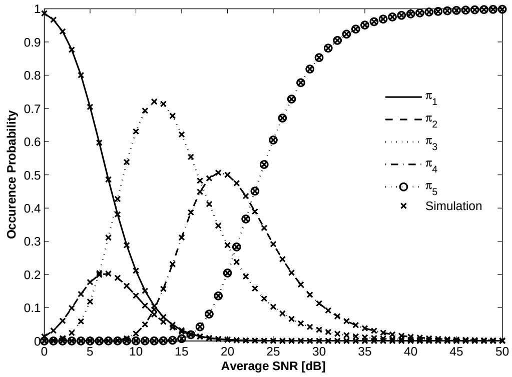

Figure 2. Occurrence probabilities of various modes, $T = 10$.

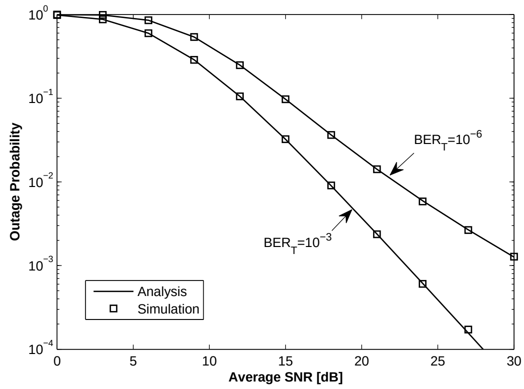

Figure 3. Outage probability for distributed SSC under adaptive modulation, $T = 10$.

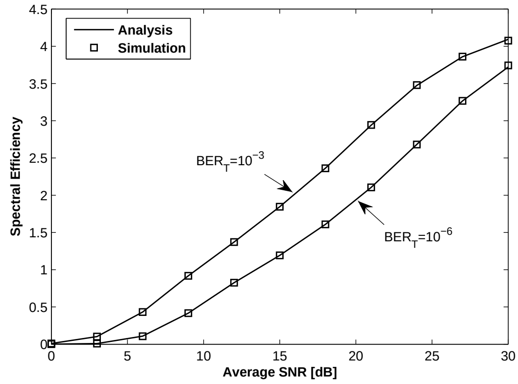

Figure 4. Spectral efficiency for distributed SSC under adaptive modulation, $T = 10$.

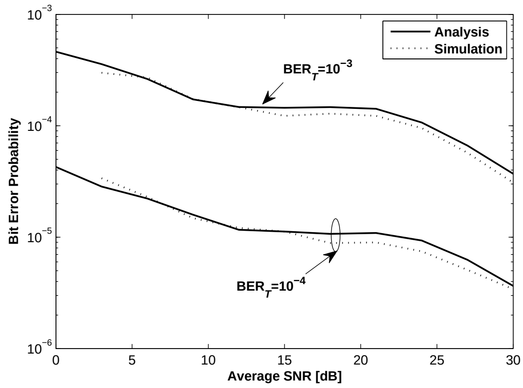

Figure 5. Bit error rate for distributed SSC under adaptive modulation, $T = 10$.

We first investigate the occurrence probability of each mode. It can be observed from Figure 2 that mode 1 (no transmit) and mode 5 (64-QAM) dominate in low and high SNR regimes, respectively. Therefore, it can be concluded that adaptive modulation is ineffective at high SNRs since the highest modulation constellation is used for most of the time.

Figure 3 shows the results of our study of the outage probability of DSSC systems with adaptive modulation as well as the impact of the target BER on the system outage probability. It can be seen that there is a good match between the analytical and simulated curves, validating the analysis in the previous section. Furthermore, in adaptive systems, the higher the required target BER is, the higher the outage probability the systems suffer. For example, with the same outage probability of 10^{-6} a system with $\mathrm{BER}_T = 10^{-6}$ gains around 8dB as compared to that with $\mathrm{BER}_T = 10^{-3}$.

Figure 4 shows the results of our investigation of the most interesting quality measure of adaptive systems, the achievable spectral efficiency. As expected, the spectral efficiency increases fastest as the average SNR increases in the middle range of SNR. It is obvious to observe that, when adaptive modulation is used, we have a trade-off between the system spectral efficiency and the error rate, e.g., for a same value of the spectral efficiency, around 5dB of transmit power can be saved since the target BER is reduced from 10^{-6} to 10^{-3}.

Figure 5 shows the results of our study of the average BER of the proposed scheme with two different values of the target BER: 10^{-3} and 10^{-4}. It can be observed that, under the fixed mode of the boundary values, the average BER is always well below the target BER and hence satisfies the QoS requirement. However, it is worth remarking that there is still a gap between the obtained BER and the target BER, implying that the spectral efficiency of the system could be further improved by optimizing the boundary values.

In Figure 6 and Figure 7, we compare the two

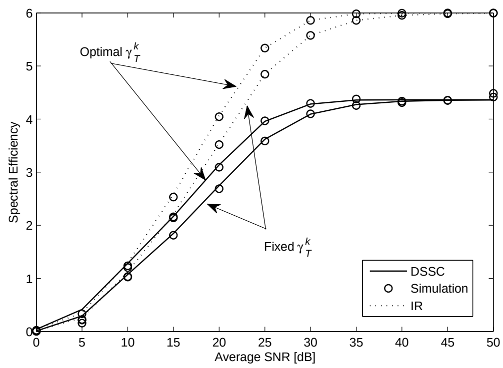

Figure 6. Bit error rates for DSSC and IR networks under adaptive modulation, $T = 10$.

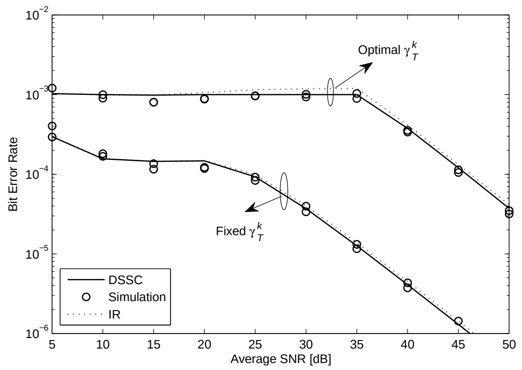

Figure 7. Spectral efficiency for DSSC and IR networks under adaptive modulation, $T = 10$.

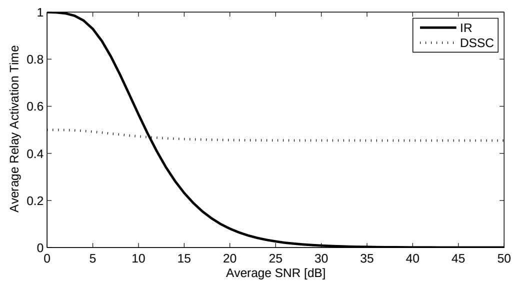

Figure 8. Average relay activation time, $T = 10$.

schemes in terms of the spectral efficiency and the bit error rate with the fixed and the optimal modes of the boundary values. Numerical results show that the performance gap in term of the spectral efficiency between the optimal and the fixed boundary values is around 3dB in the middle range of SNR. However, the gap disappears when the average SNR is high enough. It is also interesting to highlight that, under the same conditions, the IR network achieves significantly better spectral efficiency than the DSSC network at high SNRs.

Figure 8 shows the average values of the relay activation time for both schemes. Note that the average values of the relay activation time are $p_{\mathcal{R}}$ and $\Pr(\gamma_{\mathcal{D}} < T)$ for DSSC and IR networks, respectively. We can see from this figure that at lower SNRs the DSSC network experiences a low relay activation time as compared to the IR network and hence achieve higher spectral efficiency. At higher SNRs, the IR system is able to take advantage of favorable channel conditions of the direct link and transmits at the highest rates, that results in better spectral efficiency than that of the distributed SSC network while having a same average BER. Therefore, we can conclude that DSSC networks are more suitable at low SNRs while IR networks should be used at high SNRs.

5 Conclusion

We have presented our research results on the performance of distributed switch-and-stay networks under adaptive modulation. Analytic results for the occurrence probability, the outage probability, the average bit error probability and the spectral efficiency have been derived for the case of Rayleigh fading channels. These analytic results have been verified by simulation results, as shown in this paper.

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