

## Regular Article

# Jamming Impact on the Performance of MIMO Space Time Block Coding Systems over Multi-path Fading Channel

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**Abstract**– The combined effect of broad/partial band jamming on the analysis performance of multi-input/multi-output (MIMO) space time block coding (STBC) systems over Rayleigh fading channel is often neglected in the literature. To this end, this paper investigates the impact of different kinds of jamming on the performance of MIMO STBC systems. Closed form expressions for the bit error probability (BEP) of the performance of binary phase shift keying (BPSK) STBC communication system over Rayleigh fading channel under the impact of both partial band and broad-band jamming for different number of transmitting and receiving antennas are presented. Computer simulations are provided to validate the theoretical developments. It has been found that the performance of such systems is severely deteriorated in the presence of strong jamming power (with a signal to jamming power ratio less than  $-10\text{dB}$ ) and the worst scenario occurs in the presence of broad-band jamming.

## 1 INTRODUCTION

The use of multiple transmit antennas and multiple receive antennas provides the potential for high capacity communication links in multi-path fading environments, but the sensitivity of MIMO techniques to degradation by adversarial jamming must be considered in military applications. Jamming of MIMO systems has been studied previously; in [1] they proved that for a MIMO Rayleigh-fading Gaussian channel, a jammer with access to the channel input can inflict only as much damage to communication as one without access to the channel input.

Also in [2] the effect of partial band jamming on space time coded frequency hopped MIMO system has been considered. Numerical results show that space-time coded systems are robust to this type of intentional interference. Whereas, in [3] the joint effect of non-linear distortion which is produced by non-linear high power amplifiers and jammer on the performance of two transmitting (2Tx) and one receiving (1Rx) antennas of orthogonal frequency division multiplexing system in Rayleigh fading channel had been analysed. It had been shown that degradation to the system performance is expected due to the presence of jamming.

To the best of our Knowledge, closed and generalized form expression to the impact of different kinds of jamming, specially broad and partial band jamming, on the performance of BPSK MIMO STBC communication systems over Rayleigh fading channel had not been well considered. To this end, the main objective of this paper is to evaluate a closed, generalized, and simple yet accurate expression for the effect of broad and partial band jamming on the performance of BPSK MIMO STBC communication system over Rayleigh fading channel.

The paper is organized as follows. Section 2 depicts the used system model, including the jamming signal model and the channel. An analysis of the received signal is depicted in Section 3. Section 4 depicts the BER performance analysis under the impact of jamming. Numerical and simulation results are presented in Section 5. Finally, Section 6 draws the conclusions.

## 2 SYSTEM MODEL

The binary input bits to the BPSK modulator are assumed to be equi-probable and independent. The STBC encoder uses the coding scheme which was proposed by Alamouti [4] and encodes a pair of the transmitted symbols at each time instants.

Consider that we have a transmission sequence; for example  $\{x_1, x_2, x_3, \dots, x_n\}$ . Alamouti suggested that in the first time slot,  $x_1$  and  $x_2$  are sent from the first and the second antenna respectively. In the second time slot;  $x_2^*$  and  $-x_1^*$  from the first and second antenna respectively.

Note that the Alamouti codeword  $\mathbf{X}$  is a complex-orthogonal matrix, that is,

$$\mathbf{X} = \begin{pmatrix} |x_1|^2 + |x_2|^2 \end{pmatrix} \cdot \mathbf{I}_{2 \times 2} \quad (1)$$

where  $\mathbf{I}_{2 \times 2}$  denotes the  $2 \times 2$  identity matrix.

Since we use two transmitting antennas to transmit two symbols in two time slots, then the transmission rate of Alamouti code is shown to be unity and there is no change in the data rate. Denote  $h_1(n)$  as the impulse response of the Rayleigh fading channel between the first transmit antenna and the receive antenna. Let  $h_2(n)$  be the channel impulse response between the second transmit antenna and the receive antenna.

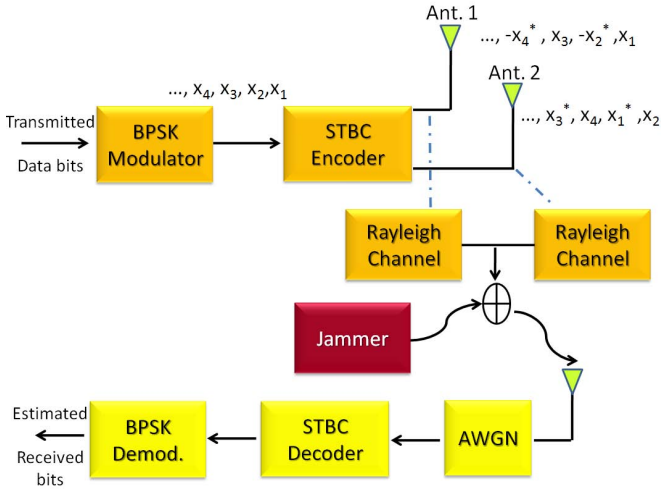


Figure 1. The BPSK STBC communication system in the presence of Jamming in Rayleigh fading channel.

The jamming signal, denoted by  $J(t)$ , is a stationary zero mean complex Gaussian process active on a fraction  $\rho$  of the signal bandwidth. The assumption of Gaussian jamming signal is considered in several articles as [5–7].

We assumed that the jammer has a flat power spectral density (psd)  $2Q_0$ . When the jammer is present, it covers a fraction  $\rho$  of the signal bandwidth  $W_s$ . In broad-band jamming, the jammer should occupy the whole communicator bandwidth. Thus, broad band jamming is considered as a special case of the partial band jamming by setting the fraction  $\rho$  to be equal to one.

Finally, all Rayleigh fading channels are assumed to be statistically independent from each other and the channel impulse responses are static for the duration of two symbols transmissions. The additive white Gaussian noise (AWGN), denoted as  $w(n)$ , represents the thermal noise and is modeled as an AWGN process with psd  $2N_0$  [8].

Figure 1 depicts the block diagram of a BPSK STBC communication system which is subjected to the impact of jamming in Rayleigh fading channels.

### 3 RECEIVED SIGNAL ANALYSIS

For the simplicity of notation, we will drop the time index. Assuming perfect synchronization, let  $y_1$  and  $y_2$  denote the received signals at time  $t$  and  $t+T_s$ , respectively, then

$$y_1 = h_1 x_1 + h_2 x_2 + J + \omega_1 \quad (2)$$

$$y_2 = -h_1 x_2^* + h_2 x_1^* + J + \omega_2 \quad (3)$$

where  $\omega_1$  and  $\omega_2$  are the additive noises of a stationary zero mean complex Gaussian process, independent of signal and with psd  $2N_0$  at time  $t$  and  $t+T_s$ , respectively.

Taking complex conjugation of the second received signal, we have the following matrix vector equation

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} J \\ J^* \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix} \quad (4)$$

Assuming perfect channel estimation scenario in which the channel gains,  $h_1$  and  $h_2$ , are exactly known to the receiver.

Multiplying both sides of equation 4 by the Hermitian transpose of the channel matrix, then

$$\begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} J \\ J^* \end{bmatrix} + \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = (|h_1|^2 + |h_2|^2) \mathbf{I}_{2 \times 2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \hat{\mathbf{J}} + \hat{\mathbf{w}} \quad (6)$$

where

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} \quad (7)$$

$$\hat{\mathbf{J}} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} J \\ J^* \end{bmatrix} \quad (8)$$

and

$$\hat{\mathbf{w}} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix} \quad (9)$$

### 4 BER PERFORMANCE ANALYSIS

In this section the BER performance of the MIMO space time block coding under the impact of jamming is analytically analyzed. Now in order to find an estimate of the transmitted symbols; we need to find the inverse of  $(\mathbf{H}^H \mathbf{H})$ .

Since  $\mathbf{H}^H \mathbf{H} = (|h_1|^2 + |h_2|^2) \mathbf{I}_{2 \times 2}$  is a diagonal matrix, the inverse is just the inverse of the diagonal elements, which can be represented as

$$(\mathbf{H}^H \mathbf{H})^{-1} = \begin{bmatrix} \frac{1}{|h_1|^2 + |h_2|^2} & 0 \\ 0 & \frac{1}{|h_1|^2 + |h_2|^2} \end{bmatrix} \quad (10)$$

Let  $\check{\mathbf{H}} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$ , then the estimate of the transmitted symbols can be evaluated as

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (\mathbf{H}^H \mathbf{H})^{-1} \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = \check{\mathbf{H}} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \check{\mathbf{H}} \begin{bmatrix} J \\ J^* \end{bmatrix} + \check{\mathbf{H}} \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix} \quad (12)$$

The noise term  $\mathbf{N} = \check{\mathbf{H}} \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix}$  is a Gaussian process with zero mean and a variance  $= E\{\mathbf{N}\mathbf{N}^H\} = 2N_0 \mathbf{I}_{2 \times 2}$ . Whereas, the jamming term  $\mathbf{Y} = \check{\mathbf{H}} \begin{bmatrix} J \\ J^* \end{bmatrix}$  is also a Gaussian process with zero mean and variance  $E\{\mathbf{Y}\mathbf{Y}^H\} = 2Q_0 \mathbf{I}_{2 \times 2}$ .

Let us compare the above system with the system with a single transmit and single receive antenna in the absence of jamming. The latter system can be modeled as

$$\mathbf{Y} = \mathbf{h}\mathbf{X} + \mathbf{n} \quad (13)$$

where  $\mathbf{X}$ ,  $\mathbf{Y}$  and  $\mathbf{h}$  are the transmitted symbol, received signal and channel fading respectively.

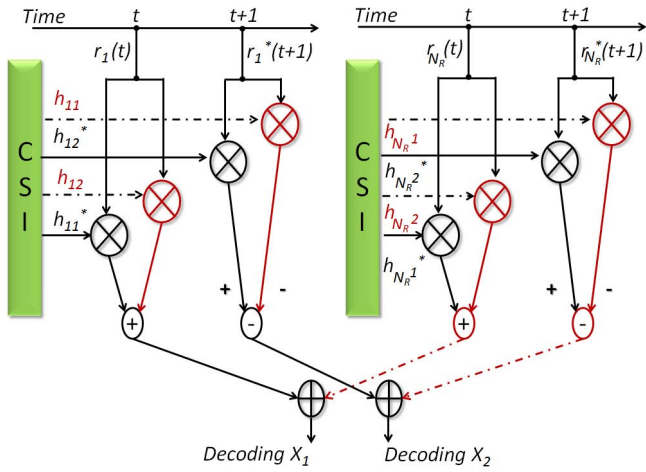


Figure 2. Alamouti STC receiver scheme for  $2 \times N_R$  MIMO.

Let  $SNR_t = \frac{E(|X|^2)}{\sigma_n^2} = \frac{E_b}{N_o}$  is defined as the SNR at the transmitter side, where  $\sigma_n^2$  is the variance of the noise term and  $E_b$  is the bit energy. Then, the conditional SNR (conditioned on the channel state information “h”) at the receiver side is

$$SNR_{siso} = |h|^2 SNR_t \quad (14)$$

For an STC  $2 \times 1$  system, it is typically required that a symbol uses the same energy as that in the uncoded system. We assume that  $E(|X_1|^2) = E(|X_2|^2) = E(|X|^2)/2$ . Then the conditioned SNR (conditioned on the channel state information  $h_1$  and  $h_2$ ) at the receiver side is

$$SNR_{2 \times 1} = \frac{|h_1|^2 + |h_2|^2}{2} \cdot SNR_t \quad (15)$$

The Alamouti STC idea for the  $2 \times 1$  MISO can be easily extended to the case of  $2 \times N_R$  MIMO, as illustrated in Figure 2

Now denote the received signals at receive antenna  $i$  as  $r_i(t)$  and  $r_i(t+1)$ , ( $i = 1, \dots, N_R$ ) respectively at time instants  $t$  and  $t+1$ . Similarly, the receiver noises at receive antenna  $i$  are denoted as  $n_i(t)$  and  $n_i(t+1)$ , ( $i = 1, 2$ ) respectively at time instants  $t$  and  $t+1$ .

Let the channel fading gain from transmit antenna  $i$  to receive antenna  $j$  be  $h_{ji}$  ( $i = 1, 2, j = 1, 2, \dots, N_R$ ). Combine the received signals in the following way [4]

$$Y_1 = \sum_{j=1}^{N_R} h_{j1}^* r_j(t) + h_{j2}^* r_j^*(t+1) \quad (16)$$

$$Y_2 = \sum_{j=1}^{N_R} h_{j2}^* r_j(t) - h_{j1}^* r_j^*(t+1) \quad (17)$$

Then, in the presence of jamming we have

$$Y_1 = \sum_{j=1}^{N_R} \left[ \sum_{i=1}^2 |h_{ji}|^2 X_1 + h_{j1}^* n_j(t) + h_{j1}^* J_j(t) + h_{j1} n_j^*(t+1) + h_{j1} J_j^*(t+1) \right] \quad (18)$$

$$Y_2 = \sum_{j=1}^{N_R} \left[ \sum_{i=1}^2 |h_{ji}|^2 X_2 + h_{j2}^* n_j(t) + h_{j2}^* J_j(t) + h_{j2} n_j^*(t+1) + h_{j2} J_j^*(t+1) \right] \quad (19)$$

It can be seen that the information symbols  $X_1$  and  $X_2$  are decoupled from each other and hence can be decoded from the combined receiver outputs  $Y_1$  and  $Y_2$  respectively in the presence of jamming.

The conditioned SNR (conditioned on the channel state information  $h_{ji}$ , ( $i = 1, 2, j = 1, \dots, N_R$ )) at the receiver side in the presence of jamming can be written as

$$SNR_{2 \times N_R} = \frac{\sum_{j=1}^{N_R} \sum_{i=1}^2 |h_{ji}|^2}{2} \cdot SNR_{tj} \quad (20)$$

$$\text{where } SNR_{tj} = \frac{E(|X|^2)}{\sigma_n^2 + Q_o}$$

Let  $N_j$  denotes the *psd* of a reference jammer that spreads the same average power  $J_{av}$ ; where  $J_{av}$  is the total average jamming power, on the total communication bandwidth that is  $N_j = J_{av}/W_s$ . Consequently,  $Q_o = J_{av}/\rho W_s = N_j/\rho$ .

Suppose that each channel undergoes independent and identically distributed (i.i.d.) Rayleigh fading, i.e.,  $h_{ji}$  are i.i.d. complex Gaussian with zero mean and unit variance.

Let  $A(h) = \sum_{j=1}^{N_R} \sum_{i=1}^2 |h_{ji}|^2$ . Then the average BER

$$P_e = \mathbb{E}_{A(h)} \left[ Q \left( \sqrt{2A(h) \frac{SNR_{tj}}{2}} \right) \right] \quad (21)$$

which can be simplified as

$$P_e = \left( \frac{1-\beta}{2} \right)^{2N_R} \sum_{k=0}^{2N_R-1} \binom{2N_R-1+k}{k} \cdot \left( \frac{1+\beta}{2} \right)^k \quad (22)$$

$$\text{where } \beta = \sqrt{\frac{SNR_{tj}}{2 + SNR_{tj}}}, \text{ and } SNR_{tj} = \frac{E_b}{N_o + N_j/\rho}.$$

## 5 NUMERICAL AND SIMULATION RESULTS

In this section, numerical examples are presented to investigate the performance of BPSK MIMO STBC under the impact of jamming in Rayleigh fading channel and validated with the aid of simulation results.

As depicted in Figure 3, the figure is plotted for SJR = 10dB, and the STBC has 2Tx antennas and 1Rx antenna for different values of  $\rho$ . The figure shows that the performance of the system decreases by increasing  $\rho$ .

For example, at  $\rho = 0.7$  and  $E_b/N_o = 15$ dB the average BEP is about  $5 \times 10^{-3}$ , while for the same value of  $E_b/N_o$  and at  $\rho = 0.1$ , the average BEP is about  $1 \times 10^{-3}$ . This figure also shows that the worst performance of the system occurs when the jamming is broadband.

This conclusion agrees with the fact that the broadband jammer is the best jammer. This fact is almost a

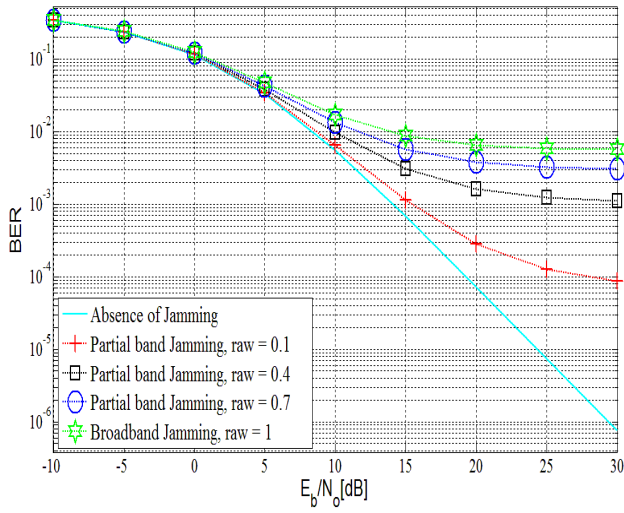


Figure 3. BER performance of BPSK Alamuti STBC Tx2-Rx1 system in the presence of Jamming in Rayleigh fading channel, SJR=10dB.

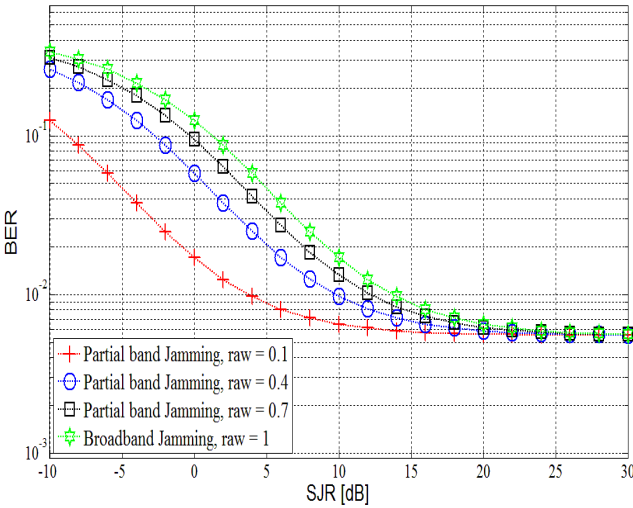


Figure 4. Impact of changing the SJR on the BER performance of BPSK Alamuti STBC 2Tx-1Rx system in Rayleigh fading channel,  $E_b/N_0=10\text{dB}$ .

universal fact and mentioned in several references such as [5].

Figure 4 depicts the effect of changing the SJR on the performance of the 2Tx-1Rx BPSK STBC system for a given  $E_b/N_0$  and for different values of  $\rho$ . It can be seen that for large SJR; the jamming effect for both types (broad and partial band) is the same. This can be explained in equation 22 as  $E_b/N_j \rightarrow \infty$  then  $E_b/N_0$  is the only factor that dominate the equation. The figure also shows that at SJR = 0dB, the broad-band jamming gives BER = 0.1244, whereas for the same value of SJR, the BER = 0.0171 for the partial band jamming.

Figure 5 depicts the impact of jamming on the BER performance of BPSK MIMO STBC 2Tx-2Rx system at SJR = -5dB. It can be seen that up to 5dB degradation on the SNR due to the presence of jamming at  $\rho = 0.1$ , and at BER =  $1 \times 10^{-2}$ , while for  $\rho = 0.7$  and for broadband jamming the system is completely jammed.

Figure 6 depicts the effect of changing the jamming

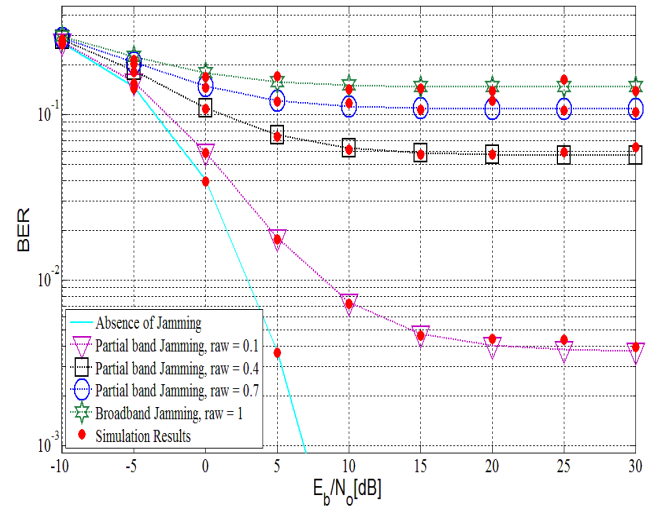


Figure 5. BER performance of BPSK MIMO STBC 2Tx-2Rx system in the presence of Jamming in Rayleigh fading channel, SJR=-5dB.

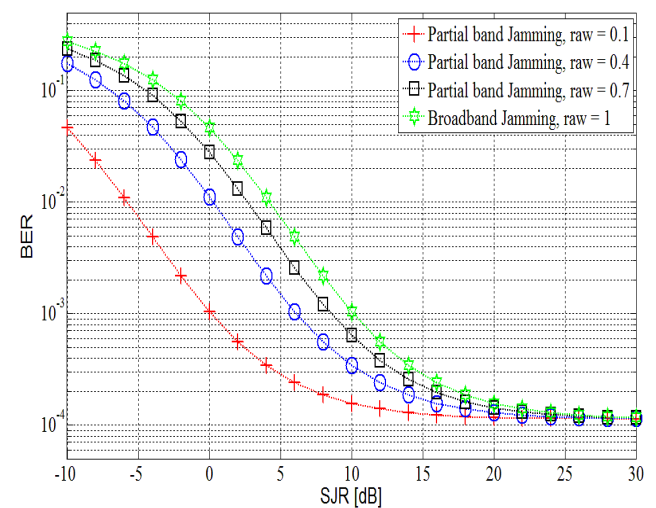


Figure 6. Impact of changing the SJR on the BER performance of BPSK MIMO STBC 2Tx-2Rx system in Rayleigh fading channel,  $E_b/N_0 = 10\text{dB}$ .

power on the performance of 2Tx-2Rx MIMO STBC system. It can be seen that the performance of the system is severely deteriorated due to the presence of jamming especially at small SJR in both partial and broad-band jamming scenarios.

In Figure 7 the effect of jamming on the BER performance of different MIMO STBC systems (2Tx-2Rx, 2Tx-3Rx, and 2Tx-4Rx) is depicted. It can be seen that for SJR = 10dB, the impact of jamming can be neglected as the jamming power is not strong enough to degrade the system performance. For SJR = 0dB, the 2Tx-4Rx system outperforms both the 2Tx-3Rx, and 2Tx-2Rx systems.

Whereas, in Figure 8, the SJR is ranging between -5dB and -10dB, it can be seen also that increasing the number of receiving antennas will enhance the system performance in the presence of jamming.

Also, it can be conclude that decreasing the signal to jamming power ratio less than -10dB can severely deteriorate the performance of the MIMO STBC systems.

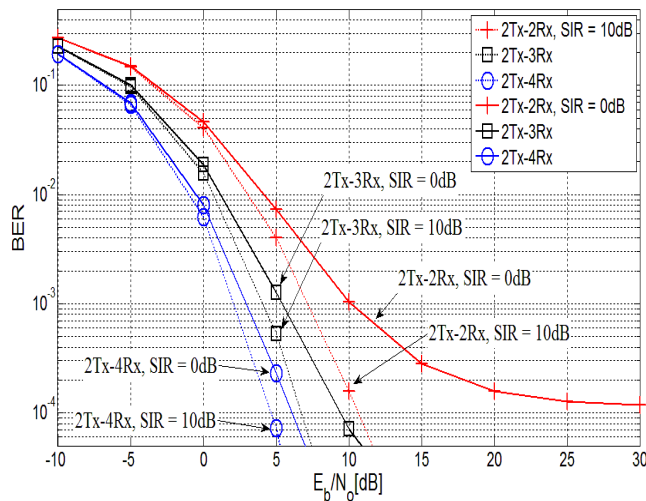


Figure 7. Effect of jamming on the BER performance of BPSK MIMO STBC system in Rayleigh fading channel with different receiving antennas,  $\rho = 0.1$ , SIR = 10 and 0dB.

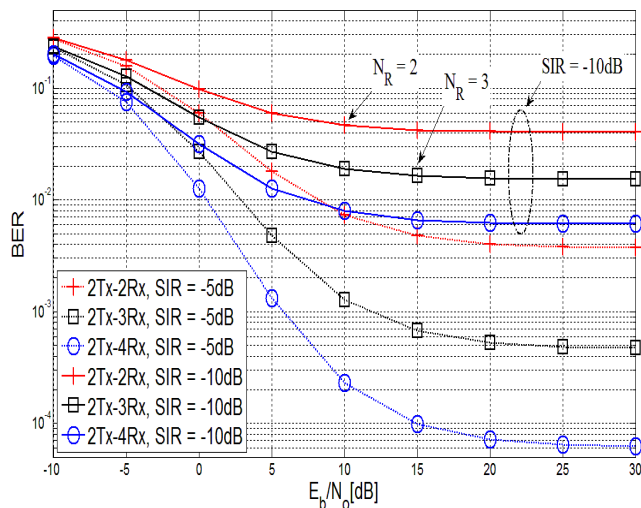


Figure 8. Effect of jamming on the BER performance of BPSK MIMO STBC system in Rayleigh fading channel with different receiving antennas,  $\rho = 0.1$ , SIR = -5 and -10dB.

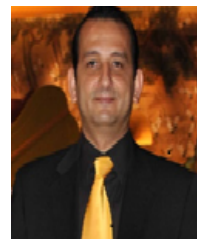
## 6 CONCLUSION

In this paper the impact of partial-band and broad-band jamming on the performance of BPSK MIMO STBC systems in Rayleigh fading channel is numerically investigated and validated with the aid of simulations. A generalized closed form expression to the performance of a BPSK MIMO STBC with different number of receiving antennas is evaluated. It has been found that the performance of the system is severely deteriorated due to the presence of jamming. It has been evaluated that the MIMO STBC system can be efficiently jammed with a signal to jamming power ratio less than -10dB.

In our future research, we will try to investigate the scenario where the jammer experienced a separate channel impulse response.

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(UWB) radio communications, spread-spectrum and secure communication signals and digital communications over fading channels.

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