*Regular Article*

# Improving DOA Estimation Accuracy Using Combination of U-Net Model and MUSIC Algorithm for Uniform Circular Arrays with Inactive Elements

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Communication: received 12 February 2025, revised 04 March 2025, accepted 10 March 2025
Online publication: 20 March 2025, Digital Object Identifier: 10.21553/rev-jec.397

*Abstract*– **In this study, a novel method is proposed to improve the accuracy of the direction of arrival (DOA) estimation for radio signal sources when a uniform circular antenna array (UCA) has inactive elements. Specifically, the full-rank covariance matrix is reconstructed by integrating a U-Net deep learning model with the multiple signal classification (MUSIC) algorithm, even with incomplete array elements. To restore essential correlation information lost due to inactive elements, a subspace-based full-rank recovery technique is employed. The reconstructed covariance matrix is then utilized by the MUSIC algorithm for accurate DOA estimation. Experimental results demonstrate significant improvements in accuracy, especially under low signal-to-noise ratio (SNR) conditions and with incomplete antenna arrays. Therefore, this approach ensures stable and precise DOA estimation even under non-ideal operating scenarios, offering a practical solution when antenna arrays experience element failures or physical obstructions.**

*Keywords*– **Direction of Arrival estimation, incomplete antenna array, deep learning, convolutional neural network.**

## 1 Introduction

Direction of arrival (DOA) estimation is a highly regarded research field due to the diversity and significance of its related applications, including radar, sonar, communications, electronic warfare, and many other areas [1]. DOA can be estimated using multiple sensors arranged in different geometric configurations. Well-known traditional methods widely applied in DOA estimation include the conventional beamforming (CB) method [2], the Minimum Variance Distortionless Response (MVDR) method [3], the signal subspace analysis methods, including the Multiple Signal Classification (MUSIC) algorithm [4], and the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) method [5]. These methods vary in computational complexity and applicability. The beamforming method is simple to implement, but experiences significant performance degradation in environments with complex noise interference and a large number of incoming sources. Subspace analysis methods can achieve high resolution in scenarios with multiple signal sources; however, they require prior knowledge of the number of sources in a predefined scenario. These methods generally assume a complete antenna array and an environment free of correlated noise between the receiving channels of the array.

The rapid advancement of Artificial Intelligence (AI) in recent years has led to groundbreaking developments in the DOA estimation problem [6]. The I/Q (In-phase/Quadrature-phase) signals from the antenna array are commonly used as input for supervised learning models to classify the angle of arrival [7]. I/Q data, which includes both the amplitude and phase information, provide a complete representation of the received signal, enabling the model to extract essential features for the determination of the angle of arrival. This approach has the advantage of being simple to implement, as it does not require complex pre-processing steps. However, its angular resolution is often limited and exhibits high errors in scenarios with multiple signal sources or when continuous angle estimation is required. Improving accuracy requires a larger training dataset to cover all classification categories, leading to increased computational costs and longer training times. An alternative approach is to use the covariance matrix of the I/Q signal as input for deep learning models [8]. This approach helps reduce data size and more effectively extract important features while leveraging correlation information between elements in the antenna array. However, in practical scenarios, antenna arrays often face various technical challenges and operate in harsh environments, leading to the failure of some sensor elements. When the antenna array has inactive elements, the size of the covariance matrix is reduced, resulting in the loss of critical information necessary for signal processing. Consequently, the accuracy of the DOA estimation for signal sources is significantly reduced, making it more challenging to analyze and locate signal sources.

In this paper, we propose a rank restoration method for the covariance matrix of an antenna array with inactive elements using a subwindow matrix approach. We develop a U-Net-based convolutional neural network model to reconstruct the covariance matrix, aiming to restore values close to the ideal state. Subsequently, traditional methods are applied to improve the accuracy of DOA estimation for signal sources. The main contributions in this paper are as follows:

- We introduce a novel rank restoration method for the covariance matrix of an antenna array with inactive elements by utilizing a subwindow matrix approach. This method effectively compensates for the inactive elements, restoring the essential correlation information between the array elements necessary for accurate signal processing.
- We develop a U-Net based convolutional neural network model to reconstruct the covariance matrix, aiming to restore it to values close to the ideal state of a complete antenna array. This deep learning approach improves the quality of the reconstructed covariance matrix, improving the overall performance of subsequent signal processing algorithms.
- By applying traditional methods, specifically the multiple signal classification (MUSIC) algorithm, to the reconstructed covariance matrix, we enhance the accuracy of direction of arrival estimation for radio signal sources. Our approach demonstrates significant improvements, particularly in scenarios where the antenna array is incomplete, thereby ensuring more reliable signal source localization even under non-ideal operating conditions.

The rest of this paper is organized as follows. Section II presents the received signal model for the UCA, both with and without inactive elements. Then, in Section III, our proposed method is introduced, where the rank restoration of the covariance matrix, the MUSIC algorithm, and the U-Net model are explained in depth. In Section IV, simulations are conducted to evaluate the proposed method, and the results are discussed. Finally, Section V concludes our work and provides directions for future research.

## 2 Received Signal Model

### 2.1 Signal model of the normal UCA

A received signal model for a uniform circular array (UCA) with a radius $R$ and $M$ elements is shown in Figure 1. Assume there are $P$ narrowband far-field signal sources that impinge on the antenna array with corresponding angles $\theta_i$, $1 \leq i \leq P$. The signal received at the output of the UCA array can be expressed as follows

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta}) \times \mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{s}(t) = \left[s_1(t), s_2(t), \ldots, s_P(t)\right]^T$ is the signal vector of the $P$ sources; $\mathbf{n}(t) = \left[n_1(t), n_2(t), \ldots, n_M(t)\right]^T$ is the uncorrelated noise vector corresponding to the $M$ receiving channels; $\mathbf{x}(t) = \left[x_1(t), x_2(t), \ldots, x_M(t)\right]^T$ is the output signal vector of the UCA; and $\mathbf{A}(\boldsymbol{\theta})$ is the array steering matrix, which is expressed as follows

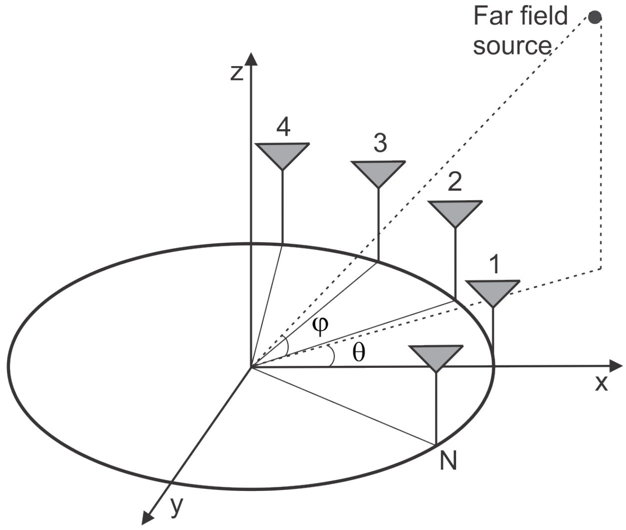



Figure 1. UCA model with $P$ signal sources.

$$\mathbf{A}(\boldsymbol{\theta}) = \left[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_P)\right]_{M \times P}. \tag{2}$$

The covariance matrix of the output signal of the UCA array is calculated using the following formula

$$\mathbf{R_{xx}} = \mathbf{x}(t)\mathbf{x}(t)^H = \mathbf{A}(\boldsymbol{\theta})\mathbf{R_{ss}}\mathbf{A}(\boldsymbol{\theta})^H + \sigma_N^2\mathbf{I_M}, \tag{3}$$

where $\mathbf{R_{ss}} = E\left[\mathbf{s}(t)\mathbf{s}(t)^H\right]$ is the correlation matrix of the transmitted signals, $\sigma_N^2$ is the noise variance, and $\mathbf{I_M}$ is the identity matrix of rank $M$.

### 2.2 Signal model of the UCA with inactive elements

Suppose that some elements in the UCA are inactive, leading to inactive signals at the array output. In this case, the set of inactive elements can be presented as follows

$$m_{\text{missing}} = \{i_1, i_2, \ldots, i_L\} \tag{4}$$

where $L$ is the number of inactive elements.

The set of active elements is presented as follows

$$m_{\text{active}} = \left\{i \,\middle|\, i \in \{0, 1, \ldots, M\}, i \notin m_{\text{missing}}\right\}. \tag{5}$$

The steering vector of the array, restricted to the active elements, is defined as follows

$$\mathbf{a}_{\text{act}}(\theta_k) = \left[e^{j\frac{2\pi}{\lambda_i}R\cos(2\pi\frac{i}{M} - \theta_k)}\right]_{i \in m_{\text{active}}}, \tag{6}$$

where $\theta_k$ is the DOA of the $k$-th signal source. The steering matrix of the array with inactive elements is given by

$$\mathbf{A}_{\text{act}}(\boldsymbol{\theta}) = \left[\mathbf{a}_{\text{act}}(\theta_1), \ldots, \mathbf{a}_{\text{act}}(\theta_P)\right]_{(M-L) \times P}, \tag{7}$$

where $P$ is the number of incoming signal sources.

## 3 Proposed Method

### 3.1 Rank restoration for the covariance matrix of the UCA with inactive elements

The covariance matrix of a normal UCA of $M$ elements has a size of $M \times M$. When the array has inactive elements, the covariance matrix is reduced in size as follows

$$\mathbf{R}_{\text{active}} = \mathbf{R}_{xx}\left[m_{\text{active}}, m_{\text{active}}\right] \in \mathbb{C}^{(M-L)\times(M-L)}. \tag{8}$$

To restore the full rank of the covariance matrix $\mathbf{R}$, the inactive elements are estimated from the active elements. The rows $i$ and columns $j$ corresponding to $m_{\text{inactive}}$ in the matrix are assigned undefined values (NaN) to represent the inactive state as follows

$$\mathbf{R}_{i,j} = \text{NaN}, \quad \forall i, j \in m_{\text{inactive}}. \tag{9}$$

The inactive elements corresponding to the value $\mathbf{R}_{i,j}$ are replaced with the local mean value $\mu_{i,j}$ of the neighboring elements as follows

$$\mu_{i,j} = \frac{1}{|N_{ij}|} \sum_{(k,l)\in N_{ij}} \mathbf{R}_{k,l}, \tag{10}$$

where $N_{ij}$ is the set of indices $(k, l)$ neighboring the inactive position $(i, j)$, defined within a subwindow matrix of size $(2h+1) \times (2h+1)$, $h$ is the window radius. $|N_{ij}|$ is the number of elements in the set $N_{ij}$, and $\mathbf{R}_{k,l}$ is the covariance value corresponding to $m_{\text{active}}$. The resized matrix $\mathbf{R}_{\text{masked}}$ is used as the input for the U-Net model for reconstruction.

### 3.2 MUSIC algorithm

The MUSIC method operates based on decomposing the covariance matrix of the received data into two orthogonal matrices, corresponding to the signal subspace and the noise subspace. The signal subspace consists of eigenvectors corresponding to the eigenvalues of the signal components, while the noise subspace contains eigenvectors representing the noise components. The DOA estimation process is performed by exploiting the properties of these subspaces, under the assumption that noise at each receiving channel is uncorrelated. The covariance matrix of the received data serves as the foundation for the entire process, which is represented as follows

$$\mathbf{R}_{xx} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_N^2 \mathbf{I}_M, \tag{11}$$

here, $\mathbf{R}_{ss} = \mathbb{E}\left[\mathbf{s}(t)\mathbf{s}(t)^H\right]$ is the correlation matrix of the transmitted signal, $\sigma_N^2$ is the noise variance, and $\mathbf{I}_M$ is the identity matrix of rank $M$. The eigenvalues of $\mathbf{R}_{xx}$ are determined by solving the following equation

$$\left|\mathbf{R}_{xx} - \lambda \mathbf{I}\right| = 0. \tag{12}$$

The eigenvector $\mathbf{V}_n$ corresponding to the specific eigenvalue $\lambda_a$ is determined from the following equation

$$\mathbf{R}\mathbf{V}_n = \lambda_a \mathbf{V}_n. \tag{13}$$

With $N$ eigenvalues, an eigenvector matrix of size $N \times N$ is formed. The eigenvectors of the noise subspace are orthogonal to the steering vectors of the array

$$\mathbf{a}^H(\theta)\mathbf{E}_N\mathbf{E}_N^H\mathbf{a}(\theta) = 0. \tag{14}$$

The MUSIC spectrum is obtained as follows:

$$P_{MUSIC} = \frac{1}{\mathbf{a}^H(\theta)\mathbf{E}_N\mathbf{E}_N^H\mathbf{a}(\theta)}, \tag{15}$$

where $\mathbf{a}(\theta)$ is the steering vector of the array, $\mathbf{E}_N$ represents the noise subspace with a size of $N \times (N-M)$.

### 3.3 U-Net model

Figure 2 illustrates the structure of the proposed U-Net model, which consists of two phases: encoding and decoding. These phases are interconnected through skip connections to maintain the integrity of information throughout the processing. In the encoding phase, the input data is the covariance matrix $\mathbf{R}_{xx}$ of size $9 \times 9$, which is processed through sequential convolutional blocks. Each block consists of a 2D convolutional layer (Conv2D) with a $3 \times 3$ filter kernel. The LeakyReLU activation function is used to reduce neuron dead zones compared to standard ReLU. After each layerconvolutional block, a MaxPooling layer with a $2 \times 2$ kernel is applied to reduce the spatial dimensions of the data while preserving essential feature information. The number of filters in the convolutional layers gradually increases from 16, 32, 64, to 128, allowing the model to learn more complex feature representations from the input matrix.

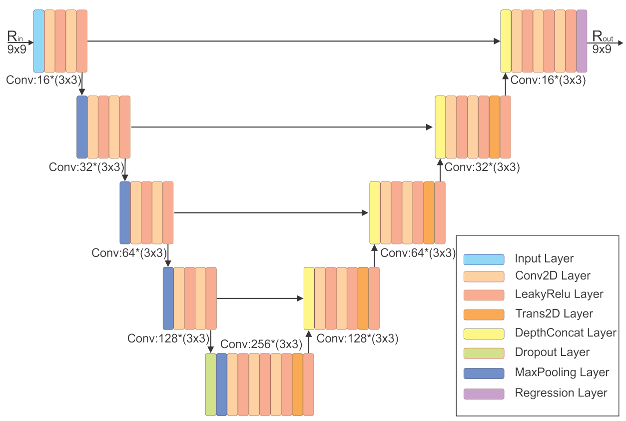



Figure 2. Architecture of the U-Net model.

The decoding phase operates in the reverse direction of the encoding phase, consisting of blocks that expand the data dimensions through transposed convolution layers, combined with regular convolutional layers where the number of kernels gradually decreases from 256 to 16 to restore the information. A key architectural feature of U-Net is the use of skip connections, where feature maps from the corresponding encoding layers are concatenated with the decoding layers through Depth Concatenation. This approach helps preserve crucial spatial information lost during dimensional reduction, while also improving the reconstruction performance of the covariance matrix. To mitigate overfitting, the model applies dropout layers with a probability of $p = 0.5$ in the deeper layers. Finally, the model employs a regression layer to reconstruct the covariance matrix to its original size, ensuring that the correlation information between antenna elements is fully restored.

The model training process was conducted over 30 epochs, applying the Adam optimizer to improve convergence ability and reduce oscillations during weight optimization. The initial learning rate was set to $10^{-4}$, ensuring a stable weight update process. The mini-batch size was set to 32, allowing the model to efficiently utilize computational resources, enhancing generalization capability, and improving convergence speed. The computational complexity of the model training process is described by the following formula

$$\text{FLOPs}_{\text{total}} = \text{FLOPs}_{\text{encoder}} + \text{FLOPs}_{\text{decoder}}. \tag{16}$$

Specifically,

$$\text{FLOPs}_{\text{total}} = \sum_{i}^{L} 2 \times H_i \times W_i \times C_{\text{in},i} \times C_{\text{out},i} \times k^2 \tag{17}$$

where $L$ is the total number of convolutional layers in the model. $H$ and $W$ represent the height and width of the input data, $C_{\text{in}}$ and $C_{\text{out}}$ correspond to the number of input and output channels of the model, and $k^2$ is the kernel size. Thus, the proposed model has a total floating point operation (FLOPs) count of 55,246,034 and was trained on an Intel(R) Core(TM) i7-6820HQ processor combined with an GPU NVIDIA Quadro M2000M.

## 4 Simulation and Results

### 4.1 Dataset

In this study, the dataset was generated according to Algorithm 1. The uniform circular antenna array model consists of 9 elements with an array radius of $R = 22.4$ m. The number of signal sources impinging on the antenna array is randomly chosen between 1 and 4 sources, with a frequency of $f_c = 6.7$ MHz and a random initial phase. The interference is assumed to be Gaussian distributed white noise, with a sample size of $N_x = 1024$. The SNR values are distributed from $-20$ dB to 20 dB, with a step size of 1 dB.

The objective of the U-Net model constructed in this study is to reconstruct $\mathbf{R}_{xx}$ in (3) into $\mathbf{R}_{\text{rct}}$ with the expectation that it does not contain noise components. This corresponds to the output signal from the antenna array reaching an ideal state, meaning it is not affected by noise. In this case, $\mathbf{x}(t)$ in (1) becomes

$$\mathbf{x}_0(t) = \mathbf{A}(\boldsymbol{\theta}) \times \mathbf{s}(t). \tag{18}$$

The reconstructed covariance matrix $\mathbf{R}_{\text{rct}}$ can be expressed as follows

$$\mathbf{R}_{\text{rct}} = \mathbb{E}\left[\mathbf{x}_0(t) \cdot \mathbf{x}_0(t)^H\right]. \tag{19}$$

The covariance matrix $\mathbf{R}_{xx}$ is in complex form, so before being fed into the model for training, it needs to be transformed into a real-valued matrix $\mathbf{R}_{in}$. Due to the properties of the Hermitian matrix, the transformation process can be described by the following formula

$$\mathbf{R}_{in} = real\{triu\{\mathbf{R}_{xx}\}\} + diag\{\mathbf{R}_{xx}\} + imag\{tril\{\mathbf{R}_{xx}\}\},$$

where the components $real\{\}$ and $imag\{\}$ are functions that extract the real and imaginary parts of a complex number, respectively. $diag\{\}$, $triu\{\}$, and $tril\{\}$ are functions that extract the main diagonal, upper triangular, and lower triangular elements of a matrix, respectively. The output matrix $\mathbf{R}_{out}$ reconstructed by the model is converted into a complex matrix $\mathbf{R}_{rct}$ before being used for DOA prediction with traditional methods.

**Algorithm 1 : Dataset Generation**

**Step 1: Parameter Setup**
- Antenna Array UCA: $M = 9$, Radius = $\lambda/2$
- Signal Sources: $P$ = Randi(1:4), $f_c = 6.7$ MHz, $N_x = 1024$
- SNR Ratio: $-20$:1:20 dB
- DOA Angles: $-179°$:180°
- Number of trials per SNR: $n = 40,000$

**Step 2: Data Generation Loop**

**for** $n = 1$ to 40,000 **do**
- Generate incoming signal: $\mathbf{s}(t)$
- Compute ideal received signal: $\mathbf{x}_0(t)$
- Compute ideal covariance matrix: $\mathbf{R}_{\text{ideal}}$
- Add Gaussian noise: $\mathbf{x}(t)$
- Compute covariance matrix: $\mathbf{R}_{xx}$

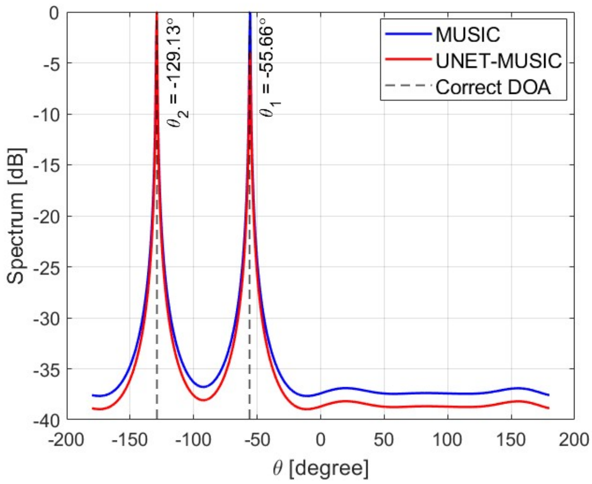
**end for**

The dataset is generated with a sample size of 40,000 for each SNR level. The total dataset consists of 1,640,000 samples of $\mathbf{R}_{in}$, which is divided into 80% for training and 20% for testing. The dataset generation, model training, and evaluation processes are implemented in Matlab 2023.

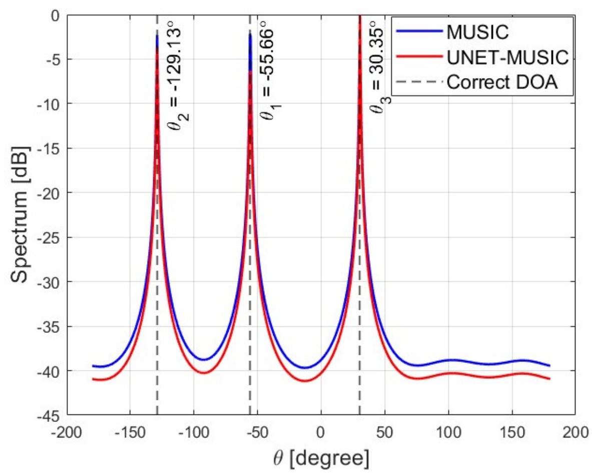
### 4.2 Evaluation of the proposed method in the Case of the normal UCA

Consider a normal UCA array with 9 elements affected by incoming signal sources at a frequency of 6.7 MHz. The interference is assumed to be Gaussian distributed white noise, with a sample size of $N_x = 1024$ at an SNR level of 1 dB. The covariance matrix $\mathbf{R}_{xx}$ of size $9 \times 9$ is used for DOA estimation using the MUSIC algorithm, while also serving as input for the U-Net network to reconstruct the ideal covariance matrix $\mathbf{R}_{rct}$. Afterward, the MUSIC algorithm is again applied to $\mathbf{R}_{rct}$ for DOA estimation.

When the UCA array has fully active elements, the results of the MUSIC spectrum (blue) and U-Net - MUSIC (red) in Figure 3 show that both methods accurately identify the spectral peaks at the actual DOA positions. In the case of two signal sources ($\theta_1 = -55.66°$ and $\theta_2 = -129.13°$), the U-Net-MUSIC algorithm maintains an accurate estimation capability while also providing a smoother spectrum and reducing background noise compared to traditional MUSIC. When the number of sources increases to three ($\theta_1 = -55.66°$, $\theta_2 = -129.13°$, $\theta_3 = 30.35°$), U-Net-MUSIC still ensures clear spectral peaks, demonstrating its ability to generalize in more complex signal scenarios. Evaluating the root mean square error (RMSE) in 1000 trials shows that, in the two-source case, MUSIC has an average RMSE of 0.1707°, while U-Net - MUSIC achieves 0.1594°, corresponding to a 6.6% improvement. When the number of sources increases, the average RMSE for MUSIC is 0.1782°, while for U-Net-MUSIC it is 0.1645°, achieving a 7.7% improvement over traditional MUSIC. Although the reduction in RMSE is not large, this still demonstrates that the use of U-Net helps reconstruct a more accurate covariance matrix, thus improving the accuracy of the MUSIC algorithm in high-noise environments (SNR = 1 dB). Notably, as the number of signal sources increases, U-Net - MUSIC still proves effective, confirming its stability and adaptability in DOA estimation problems.




(a) Case with two incoming signal sources




(b) Case with three incoming signal sources

Figure 3. DOA Estimation Results Using MUSIC and U-Net-MUSIC.

Figure 4 presents a comparison of the root mean square error (RMSE) between the traditional MUSIC algorithm and the U-Net-MUSIC method in cases of two and three signal sources, with the SNR level varying from $-10$ dB to 20 dB, based on 1000 experimental trials. The results show that when SNR $< 0$ dB, the RMSE of both methods is high due to the strong impact of noise on the DOA estimation process. However, U-Net-MUSIC consistently has a lower RMSE than MUSIC, reflecting its ability to more accurately reconstruct the covariance matrix using deep learning, significantly reducing the effect of noise. When SNR increases from 0 dB to 10 dB, RMSE decreases significantly for both methods, but U-Net - MUSIC still maintains a lower error rate than MUSIC, confirming its effectiveness. When SNR $> 10$ dB, the RMSE of both algorithms nearly converges to a very low and similar value. When the number of incoming signal sources increases, the RMSE of both methods is higher than in the two-source case, reflecting the increased complexity of the DOA estimation problem. However, U-Net-MUSIC still maintains an advantage over traditional MUSIC across the entire low SNR range, demonstrating that using the U-Net network to reconstruct the covariance matrix significantly improves estimation performance, even when the number of sources increases. rom the experimental results, it can be concluded that U-Net-MUSIC provides a clear advantage over traditional MUSIC, especially in low SNR conditions and with multiple signal sources, where its ability to suppress noise and accurately reconstruct information significantly reduces estimation errors.

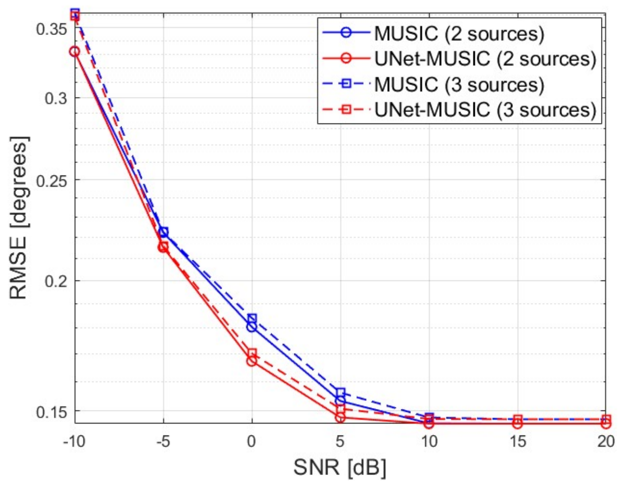
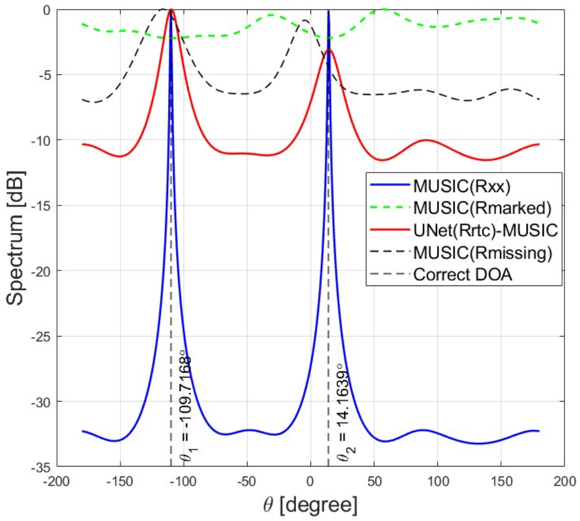


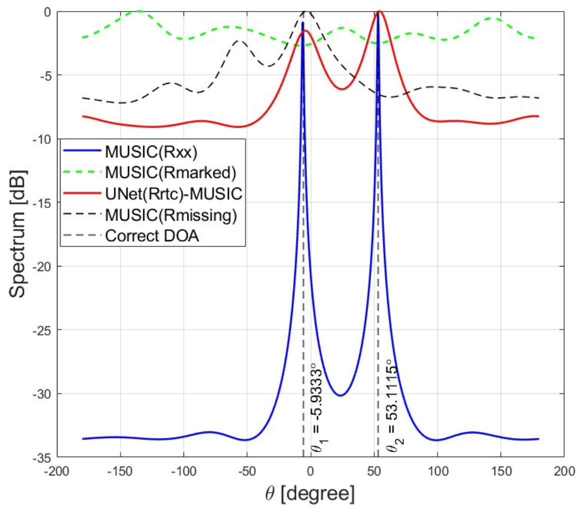

Figure 4. RMSE (degrees) of the methods when the number of incoming sources is 2 and 3.

### 4.3 Evaluation of the proposed method in the case of the UCA with inactive elements

When the UCA array has inactive elements, the quality of the DOA estimation degrades significantly due to the loss of spatial information, which directly affects the accuracy and completeness of the covariance matrix, as well as the performance of the MUSIC algorithm. The results of the inactive element scenarios in Figure 5 show that when the MUSIC algorithm is applied directly to the covariance matrix with inactive elements (MUSIC($\mathbf{R}_{\text{inactive}}$)), the DOA spectrum is severely distorted, showing spurious peaks and reduced angular resolution. Restoring the covariance matrix size using the subwindow-based method (MUSIC($\mathbf{R}_{\text{masked}}$)) and then applying MUSIC to this matrix still fail to accurately reconstruct the true DOA positions, especially as the number of inactive elements increases.



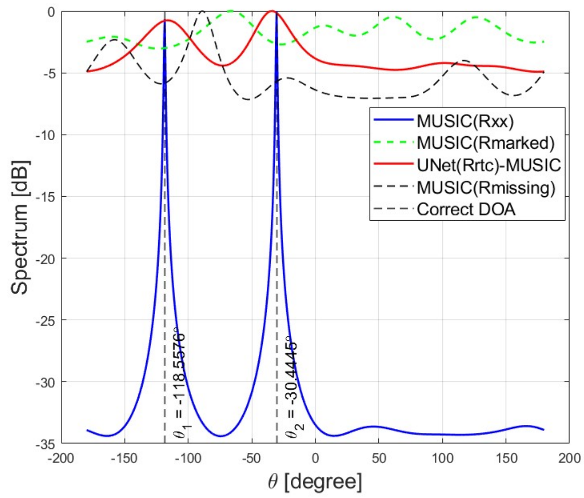

(a) UCA with inactive element number 7



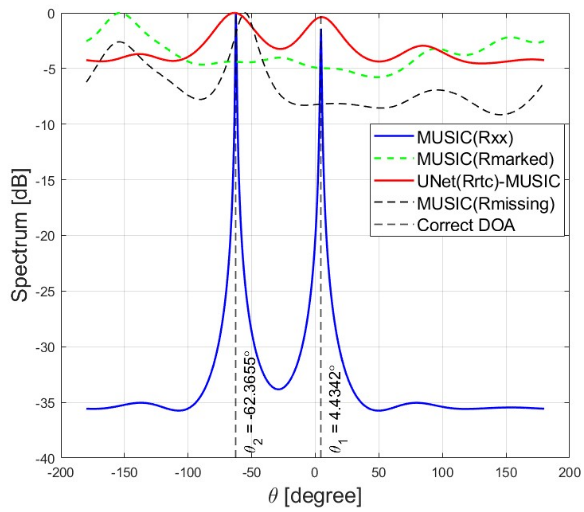

(b) UCA with inactive elements number 2 and 7

Figure 5. DOA estimation results at SNR = 5 dB in cases of inactive elements in the UCA.

In this context, the U-Net-MUSIC method demonstrates superior effectiveness in reconstructing the covariance matrix and improving the quality of DOA estimation. Instead of directly using the incomplete matrix, the U-Net model is trained to reconstruct the ideal covariance matrix $\mathbf{R}_{\text{rct}}$ from the marked matrix $\mathbf{R}_{\text{masked}}$, significantly restoring lost spatial information through the subwindow-based method. When the MUSIC algorithm is applied to this reconstructed matrix (U-Net($\mathbf{R}_{\text{rct}}$) - MUSIC), the resulting DOA spectrum closely resembles the MUSIC($\mathbf{R}_{xx}$) spectrum in the case of a fully populated array, with significantly sharper and more accurate peaks compared to MUSIC($\mathbf{R}_{\text{inactive}}$) and MUSIC($\mathbf{R}_{\text{masked}}$). Notably, as the number of inactive elements increases, the performance of the traditional MUSIC method significantly declines. However, the U-Net-MUSIC method still maintains its ability to accurately determine DOA positions. This demonstrates that the U-Net network is capable of learning and effectively reconstructing the covariance matrix structure, reducing estimation errors in DOA estimation when the UCA array has inactive elements.




(a) UCA with inactive elements 2, 3, and 4




(b) UCA with inactive elements 2, 5, and 7

Figure 6. DOA estimation results for UCA with three inactive elements at SNR = 5 dB.

The positions of inactive elements in the UCA array significantly affect the performance of DOA estimation, especially for the MUSIC algorithm, which relies on the covariance matrix. In Figure 6, it can be observed that when the inactive elements are adjacent to each other (inactive elements 2, 3, and 4), the geometric structure of the array becomes locally unbalanced, leading to a significant decrease in the angular resolution. The MUSIC spectrum in this case shows that the MUSIC algorithm applied to the incomplete matrix MUSIC($\mathbf{R}_{\text{inactive}}$) suffers severe distortion, with multiple spurious peaks and an inability to accurately determine the actual DOA positions. Meanwhile, the U-Net-MUSIC method demonstrates outstanding performance in reconstructing the spectrum, closely matching MUSIC($\mathbf{R}_{xx}$), effectively reducing noise and improving the sharpness of the spectral peaks. However, the spectral width remains larger than in the ideal case, indicating that the impact of adjacent inactive elements has not been entirely eliminated.

In contrast, when inactive elements are distributed randomly across the array (for example, inactive elements 2, 5, and 7), the impact of data loss is more evenly dispersed, reducing localized disruptions to the overall structure of the array. In this scenario, the MUSIC spectrum using the incomplete matrix MUSIC($\mathbf{R}_{\text{inactive}}$) still suffers from noise but is less distorted compared to the case of adjacent inactive elements. Meanwhile, MUSIC($\mathbf{R}_{\text{masked}}$) shows significant improvement, reflecting a better ability to recover inactive data when the loss is not concentrated in a specific region. Notably, U-Net-MUSIC continues to demonstrate superior performance, producing a reconstructed spectrum with sharper peaks, reduced background noise, and better alignment with the true DOA positions than the other methods. This indicates that when data loss is scattered, deep learning models can effectively exploit spatial features to reconstruct inactive information, thereby enhancing the accuracy of the MUSIC algorithm.

From the above results, it can be seen that the U-Net-MUSIC method consistently improves estimation performance, especially in cases where inactive elements are scattered. In such scenarios, the remaining spatial information in the array is still sufficient to support the reconstruction of the covariance matrix. These findings confirm the potential of integrating deep learning with MUSIC to overcome the limitations caused by data loss in real-world antenna systems. In addition, they open new research directions for optimizing reconstruction models to better accommodate the specific characteristics of missing data scenarios.

Figure 7 presents the RMSE results of the U-Net-MUSIC algorithm when the UCA array has between 1 and 3 missing elements, under varying SNR conditions from $-15$ dB to 20 dB. The statistical results are based on 1000 trials, with the number of random signal sources ranging from 1 to 2. Analysis of the results shows that when only one element is missing, the U-Net-MUSIC algorithm still maintains high accuracy, with RMSE nearly equivalent to the fully populated case. This demonstrates that the method can effectively reconstruct the covariance matrix when the level of data loss is minimal. However, as the number of missing elements increases (2 or more), especially under low SNR conditions ($< -5$ dB), the RMSE increases significantly, reflecting severe degradation of spatial information, which negatively impacts DOA estimation quality. When SNR improves ($> 0$ dB), RMSE in these cases decreases significantly and stabilizes, even when multiple elements are missing. Overall, U-Net-MUSIC demonstrates the ability to reconstruct missing information when only a few elements are lost, contributing to improved system accuracy. However, when the number of missing elements exceeds a certain threshold ($\geq 3$ elements), performance declines due to excessive data loss. Therefore, further research on optimizing rank restoration methods for covariance matrices is crucial to maintaining stable accuracy in scenarios where multiple elements are missing.

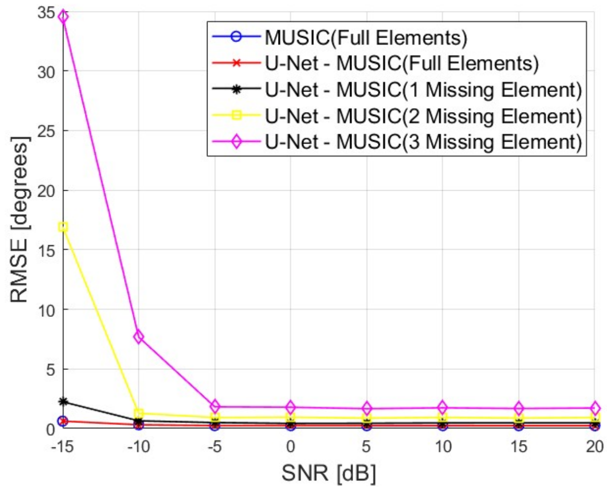



Figure 7. RMSE Results of the U-Net - MUSIC Method for UCA with Missing Elements.

An important criterion for evaluating the performance of DOA estimation methods is the computational time of the system. Table 1 presents the processing time of DOA methods over 1000 trial runs when applied to different covariance matrices, particularly in scenarios where the UCA array has missing elements.

Table I
**DOA Estimation Time**

| **Estimation Method** | **Time (ms)** |
|---|---|
| MUSIC ($\mathbf{R}_{xx}$) | 3.2 |
| MUSIC ($\mathbf{R}_{\text{masked}}$) | 3.2 |
| MUSIC ($\mathbf{R}_{\text{missing}}$) | 3.1 |
| U-Net($\mathbf{R}_{\text{rct}}$) - MUSIC | 24.16 |

The results show that the MUSIC algorithm, when applied to the full covariance matrices $\mathbf{R}_{xx}$ and $\mathbf{R}_{\text{masked}}$, has the same processing time of 3.2 ms. When using the matrix $\mathbf{R}_{\text{missing}}$, where the matrix size is reduced according to the number of missing elements, the computational time decreases to 3.1 ms. For the U-Net($\mathbf{R}_{\text{rct}}$)-MUSIC method, the processing time increases significantly to 24.16 ms. Although this method improves the accuracy of DOA estimation, the high computational cost can become a limitation for real-time applications. Therefore, selecting the appropriate DOA estimation method depends on the specific requirements of the application. If speed is the priority, the traditional MUSIC algorithm remains the optimal choice. However, in cases where accuracy is more critical, using the U-Net-MUSIC approach offers significant advantages.

# 5 Conclusion

This paper proposes a method that combines the U-Net convolutional neural network with the MUSIC algorithm to improve the accuracy of direction of arrival estimation in uniform circular antenna arrays with inactive elements. This approach utilizes rank restoration techniques to reconstruct a complete covariance matrix, followed by applying the MUSIC algorithm to determine the direction of the incoming signal sources. Experimental results demonstrate that U-Net effectively restores information lost due to inactive elements, reducing distortions in the MUSIC spectrum, minimizing background noise, and improving DOA estimation accuracy even under low SNR conditions. When the number of inactive elements increases, the proposed method maintains a more stable performance compared to traditional MUSIC. Notably, when inactive elements are randomly distributed, the reconstruction of the covariance matrix becomes even more effective. This method not only overcomes the limitations of MUSIC in incomplete arrays but also opens new research directions for future applications, expanding models with more advanced network architectures to further enhance accuracy and generalization capability in real-world antenna systems. However, the effectiveness of applying this method depends on the specific requirements of the application, particularly the computational cost. Although U-Net-MUSIC improves estimation accuracy, its higher processing time can be a drawback in systems that require real-time performance. Therefore, selecting the optimal method requires balancing accuracy and computational feasibility for the specific application.

The research results provide a foundation for extending applications to more complex problems, especially in nonuniform, adaptive, or randomly positioned antenna arrays, where missing or displaced elements impact DOA estimation performance. The proposed method also has potential applications in challenging environmental conditions or low SNR scenarios, improving accuracy compared to traditional MUSIC algorithms. Furthermore, further research could integrate deep learning models to enhance generalization and robustness. Future applications include radar, wireless communication, sonar, and next generation networks (6G), where accurate direction estimation is crucial for positioning and tracking moving sources. Ultimately, deploying this model in practical applications requires careful consideration of the trade-off between accuracy and real-time performance.


## Acknowledgment

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 102.04-2021.14.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