On the Double Doppler Effect Generated by Scatterer Motion

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Abstract— In a time-varying transmission channel, the received signals are subject to frequency shifts due to the Doppler effect. The Doppler frequency is dependent on the carrier frequency and channel variation rate. In a fixed wireless channel, the channel variations are caused by scatterer motion. In this paper, we investigate analytically the Doppler effects generated by scatterer motion under different scatterer velocity distributions using the ring-of-scatters geometric model. The proposed model considers Doppler frequency components caused by scatterer mobility to both received and reflected signals at each scatterer, and therefore is called the double Doppler model. The analytical curves are compared and statistically tested with several measurement results published in the literature. At low scatterer speeds, e.g., generated by moving foliage, the exponential velocity distribution is an appropriate model to describe the time-varying nature of the fixed wireless channels. The curve fitting results also show that our analytical model better approaches the empirical curves than the single Doppler model does. However, further investigation is still needed to find a suitable scatterer velocity distribution that closely describes the double Doppler effect in fast-variation fixed wireless channels, e.g., caused by passing vehicles.

Keywords— Double Doppler effect, Doppler power spectrum, scatterer motion, scatterer velocity distribution.

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1 Introduction

In wireless communications, the Doppler phenomenon is caused by the varying nature of the propagation environment [1, 2]. The Doppler shift is mostly assumed to be generated by subscriber motion. A receiver traveling at a speed of \( v \) m/s will cause a Doppler shift \( \nu = \frac{v_p \cos \alpha}{c} \) to carrier frequency \( f_c \), where \( v_p \) is the signal propagation velocity, and \( \alpha \) is the angle between the receiver traveling direction and the receiver-transmitter axis. In a NLOS environment, if the directions of arrival at the subscriber are uniformly distributed over \([0, 2\pi]\) and if the multipath components arrive at the same time with equal power, the Doppler power spectral density, due to subscriber motion, follows Clarke’s famous model and has the well-known \( U \)-shape [1, 3–5].

While Clarke’s model is a popular tool to describe the Doppler phenomenon caused by subscriber mobility, we cannot use this model for fixed wireless channels. For fixed wireless communications applications, e.g., the Multichannel Multipoint Distribution Service (MMDS) system, even if the subscriber and the transmitter are stationary, the received signals still experience Doppler shifts. This phenomenon was studied empirically in [6–8] but no analytic models were presented to support the measurement results until the publication of [9] and [10]. In [9], the problem was studied analytically and a channel model for the indoor channel was proposed. In this model, the scatterers are assumed either stationary or mobile at a fixed velocity or at uniformly distributed velocities. While this model is suitable for indoor environments, where the scatterers are either stationary, e.g., furnitures and walls, or slowly mobile at similar velocities, e.g., pedestrians, it is invalid for outdoor channels, where mobile scatterers have different velocities.

Using a different approach, Roy et al. [10], assumed that the subscribers are moving at a constant speed while the scatterers are moving at random velocities with different velocity distributions. The classic ring-of-scatterer geometric model was considered. The Doppler shift of a NLOS multipath component, reflected from a scatterer, is the summation of two Doppler frequencies. The first component is caused by the scatterer mobility with respect to the transmitter and the second one is due to the receiver mobility relative to the scatterer. To validate the model, the measured Doppler power spectra presented in [7] were reproduced and fitted to the analytical model. The model validation shows that the scatterer velocity randomness is successfully addressed and incorporated in the model of Roy. However, the Doppler frequency component caused by the scatterer mobility with respect to the receiver was not taken into account.

In this paper, we extend the research results presented in [10] by investigating the double Doppler phenomenon generated by scatterer mobility itself in...
a fixed wireless channel using the ring-of-scatterers geometric model. In the proposed model, both Doppler frequency components, caused by the scatterer mobility with respect to the transmitter and the receiver to the received and reflected signals, respectively, at a scatterer, are addressed and investigated in depth. The analytical model is studied under different scatterer velocity distributions, including uniform, Gaussian, exponential and triangular distributions. The proposed model is validated through curve fitting and statistical analysis with empirical curves, reproduced from the results published in [6–8].

The remainder of the paper is organized as follows. Section 2 presents the calculation of double Doppler frequencies caused by scatterer mobility. In Section 3, the autocorrelation functions and the Doppler power spectra are derived for the ring-of-scatterer geometric model under different velocity distributions. Curve fitting and statistical tests are presented in Section 4. Section 5 concludes the paper.

2 Doppler Shift Caused by Scatterer Motion

Consider a fixed wireless propagation channel where transmitted signals are emitted from the transmitter, reflected by a scatterer and received at the receiver. As the scatterer is assumed to be moving with respect to the transmitter, the signals received at the scatterer experience a Doppler frequency shift \( \nu_{TS} \) given by:

\[
\nu_{TS} = -f_c \times \frac{v_{Sx}}{v_p} \cos \alpha_{TS},
\]

where \( v_{Sx} \) is the scatterer velocity and \( \alpha_{TS} \) is the angle between the scatterer velocity vector, \( \vec{v}_{Sx} \), and the scatterer-transmitter axis.

The signal received at the scatterer is then reflected to the receiver. The carrier frequency of the reflected signal is \( f_c + \nu_{TS} \). Due to the motion of the scatterer with respect to the receiver, the signals reflected from the scatterer and received at the receiver are affected by an additional Doppler shift \( \nu_{SR} \), calculated by:

\[
\nu_{SR} = \frac{(f_c + \nu_{TS}) \times v_{Sx}}{v_p} \cos \alpha_{SR},
\]

where \( \alpha_{SR} \) is the angle between the scatterer velocity vector and the scatterer-receiver axis.

The ratio between the scatterer speed, \( v_{Sx} \), and the signal propagation velocity, \( v_p \), is very small, usually on the order of \( 10^{-7} \). Doppler frequency \( \nu_{TS} \) is on the order of tens of Hz, and thus negligible compared to the carrier frequency, \( f_c \). (2) is rewritten as follows:

\[
\nu_{SR} \approx \frac{f_c \times v_{Sx}}{v_p} \cos \alpha_{SR}.
\]

The total Doppler shift caused by the scatterer motion to the signals received at the receiver is the sum of the two Doppler components, \( \nu_{TS} \) and \( \nu_{SR} \), calculated in (1) and (3), i.e.

\[
\nu = \nu_{TS} + \nu_{SR}.
\]

The Doppler shift caused by the scatterer motion with respect to the transmitter, \( \nu_{TS} \), was discussed and characterized in [10]. The double Doppler components, \( \nu_{TS} \) and \( \nu_{SR} \), were analytically investigated in [9] for indoor channels with mixed stationary and mobile scatterers at fixed velocities. The empirical Doppler power spectra caused by scatterer mobility in fixed wireless channels were presented in [6–8, 11, 12]. In the subsequent sections, we investigate the double Doppler effects for the fixed wireless channel with mobile scatterers under different scatterer velocity distributions.

3 Doppler Power Spectral Density

3.1 Geometry Model

We consider the ring-of-scatterers geometric model [3, 10] in order to investigate the double Doppler phenomenon. As illustrated in Figure 1, the transmitter-receiver separation is \( d \). The mobile scatterers are uniformly distributed on a circle of radius \( r \) around the receiver. In order to guarantee that the attenuation on each multipath component is independent of path length, the radius \( r \) needs to be small in comparison to the distance \( d \): \( r \ll d \). There is no direct component received at the receiver from the transmitter. The multipath components are supposed to have equal power and the multipath excess delay differences are assumed not greater than the time resolution of the receiver.

The transmitter-receiver axis forms an angle \( \delta_0 \) with the \( x \) axis. The angle between the transmitter-receiver axis and the transmitter-scatterer axis is denoted \( \delta \). Since the scatterers are uniformly distributed on a circle, the angle between the transmitter-receiver axis and the receiver-scatterer axis, \( \alpha \), is uniformly distributed between \( 0 \) and \( 2\pi \). The transmitter, \( Tx \), and the receiver, \( Rx \), are located at the center of a sphere of radius \( r \).
Rx, are stationary. The scatterers, Sx, are mobile. The direction γsx of the scatterer velocity vectors is also uniformly distributed between 0 and 2π. The double Doppler effect will be investigated under different scattering velocity distributions, including Gaussian, exponential, uniform and triangular distributions.

### 3.2 Autocorrelation Function

Following the same reasoning used in [3, 13, 14], the received carrier from the nth scatterer is given by:

\[
S_n(t) = C_n \exp(j(2\pi(f_c + \nu_n)t + \theta_n)),
\]

where \(C_n\) is the attenuation; \(\nu_n\) is the compound Doppler shift caused by the transmitter, scatterer and receiver, if any, and \(\theta_n\) is the total phase shift. These parameters are calculated on the transmitter–scatterer–receiver path.

The received carrier at the receiver at passband is the summation of reflected signals from all scatterers, as follows:

\[
s(t) = \sum_{n=1}^{N} C_n \exp(j(2\pi f_c + \nu_n)t + \theta_n),
\]

where \(N\) is the number of mobile scatterers.

The baseband representation of the received carrier is:

\[
c(t) = \sum_{n=1}^{N} C_n \exp(j(2\pi f_c + \nu_n)t + \theta_n),
\]

where

\[
T_c(t) = \sum_{n=1}^{N} C_n \cos(2\pi f_c + \nu_n)t + \theta_n),
\]

\[
T_s(t) = \sum_{n=1}^{N} C_n \sin(2\pi f_c + \nu_n)t + \theta_n).
\]

The autocorrelation of the received carrier at baseband is:

\[
R(\tau) = \langle c(t)c^*(t - \tau) \rangle = \langle (T_c(t) + jT_s(t))(T_c(t - \tau) - jT_s(t - \tau)) \rangle,
\]

where \(\langle \cdot \rangle\) is the averaging operator.

Following [14], pp. 77-78, the \(\langle (T_c(t)T_c(t - \tau) \rangle\) term can be expressed as:

\[
\langle (T_c(t)T_c(t - \tau) \rangle = \langle T_c(t)T_c(t - \tau) \rangle = \langle T_s(t)T_s(t - \tau) \rangle.
\]

Replacing (10) in (9), the autocorrelation \(R(\tau)\) is:

\[
R(\tau) = 2 \langle (T_c(t)T_c(t - \tau) \rangle - 2j \langle (T_c(t)T_s(t - \tau) \rangle
\]

\[
= 2 \left[ \frac{1}{2} E_0 \sum_{n=1}^{N} \cos(-2\pi \nu_n \tau) \right] - 2 \left[ \frac{1}{2} E_0 \sum_{n=1}^{N} \sin(-2\pi \nu_n \tau) \right]
\]

\[
= E_0 \sum_{n=1}^{N} \langle e^{2\pi \nu_n \tau} \rangle,
\]

where \(E_0\) is the signal magnitude, i.e., square root of the signal power: \(E_0^2 = \sum_{n=1}^{N} C_n^2\).

When \(N\) tends towards infinity, the summation in (9) becomes an integral on the circle of radius \(r\), i.e., on the continuous variable \(\alpha\) from 0 to 2π:

\[
R(\tau) = \frac{1}{2\pi} E_0^2 \int_0^{2\pi} \langle e^{2\pi \nu \tau} \rangle d\alpha.
\]

Replacing the corresponding Doppler components, \(\nu\), into (12), we obtain the autocorrelation function, and therefore, the Doppler power spectrum.

### 3.3 Doppler Spectra Generated by Scatterer Motion

Utilizing the ring-of-scatterers geometric model in Figure 1, the angles \(\alpha_ts\) and \(\alpha_sr\) are derived as:

\[
\alpha_ts = \delta_t + \delta - \gamma_s,
\]

\[
\alpha_sr = \alpha + \delta_t - \gamma_s.
\]

The Doppler frequency caused by scatterer motion is:

\[
\nu_s = \nu_ts + \nu_sr = \frac{f_c}{v_p} \left[ \cos(\delta_0 + \delta - \gamma_s) + \cos(\alpha + \delta_t - \gamma_s) \right]
\]

\[
= \frac{2 f_c}{v_p} \cos \left( \frac{\delta + \alpha}{2} + \delta_t - \gamma_s \right) \cos \left( \frac{\delta - \alpha}{2} \right).
\]

The autocorrelation function of the received carrier complex envelope is represented in (15). Reducing the integral on variable \(\gamma_s\) gives:

\[
R(\tau) = \frac{1}{2\pi} E_0^2 \int_{\gamma_s}^{2\pi} \int_0^\infty \left[ 4 \frac{f_c}{v_p} \cos \left( \frac{\delta - \alpha}{2} \right) \right] p_{\nu_s}(\nu_s) d\nu_s.
\]

Next, the integral on variable \(\alpha\) is reduced. One observes that, as \(\alpha\) varies from 0 to 2\(\pi\), \(\delta - \alpha\) varies from 0 to \(\pi\). Applying (6.681.5) in [19], pp. 724 gives:

\[
R(\tau) = \int_{\gamma_s}^{2\pi} \int_0^\infty \left[ 2 \frac{f_c}{v_p} \cos \left( \frac{\delta + \alpha}{2} + \delta_t - \gamma_s \right) \cos \left( \frac{\delta - \alpha}{2} \right) \right] p_{\nu_s}(\nu_s) d\nu_s.
\]

One observes that the autocorrelation, and therefore the Doppler spectrum, is dependent on the scatterer velocity distribution. If the scatterer velocity is constant, i.e., \(\nu_s = \nu_{\text{max}}\), performing the Fourier transform on the above autocorrelation function, one obtains:

\[
S(\nu) = \frac{1}{2\pi} E_0^2 G_{\nu_{\text{max}}}^{20} \left[ \frac{\nu^2}{4\nu_{\text{max}}^2} \right] \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0 \right],
\]

where \(G_{\nu_{\text{max}}}^{20} \left[ x | a_1, \ldots, a_p, b_1, \ldots, b_q \right] \) represents the MeijerG function and \(\nu_{\text{max}}\) is the maximum Doppler frequency.
Doppler frequency induced by scatterer motion on the reflected signal, i.e., $v_{\text{max}} = \frac{\nu_0}{c} \nu_{\text{max}}$.

In the following Sections, we analytically derive the autocorrelation functions and the Doppler power spectral densities for uniform, exponential, Gaussian and triangular scatterer velocity.

3.3.1 Uniform Distribution: The scatterer velocity is uniformly distributed from 0 to $v_{\text{max}}$. The probability density function (pdf) is:

$$p(v_{\text{SG}}) = \begin{cases} \frac{1}{v_{\text{max}}}, & 0 \leq v_{\text{SG}} \leq v_{\text{max}}, \\ 0, & \text{otherwise}, \end{cases}$$

where $\mu_{v_{\text{SG}}} = \frac{v_{\text{max}}}{2}$ and $\sigma_{v_{\text{SG}}} = \frac{v_{\text{max}}}{2\sqrt{3}}$ are the mean and standard deviation of the scatterer velocity, respectively. The autocorrelation function then becomes:

$$R(\tau) = \frac{1}{v_{\text{max}}} \int_0^{v_{\text{max}}} \int_0^{v_{\text{max}}} \frac{2\pi f_c v_{\text{SG}}}{v_p} R_{\frac{v_{\text{SG}}}{v_p}}(\tau) dv_{\text{SG}} = 2F_3 \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}; 1, 1, \frac{3}{2}; -\frac{(2\pi f_c v_{\text{max}})^2}{v_p^2} \right) \right)$$

where $p_{F_3}(a_1, \cdots, a_p; b_1, \cdots, b_q; x)$ is the generalized hypergeometric function. The corresponding Doppler power spectrum is:

$$S(v) = \frac{G_{33}^0 \left( \frac{v^2}{4
u_{\text{max}}^2}, \frac{1}{2}, \frac{1}{2}, 1, 0, 0, 0 \right)}{4\nu^2 v_{\text{max}}^2}.$$  

3.3.2 Exponential distribution: The scatterer velocity is exponentially distributed with mean velocity $v_0$ and the pdf is:

$$p(v_{\text{SG}}) = \begin{cases} \frac{1}{v_0} e^{-\frac{v_{\text{SG}}}{v_0}}, & v_{\text{SG}} \geq 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\mu_{v_{\text{SG}}} = v_0$ and $\sigma_{v_{\text{SG}}} = v_0$. The corresponding autocorrelation function is:

$$R(\tau) = \frac{1}{v_0} \int_0^{v_0} \int_0^{v_0} \frac{2\pi f_c}{v_p} e^{-\frac{v_{\text{SG}}}{v_0}} dv_{\text{SG}} = 2F_2 \left( \frac{\nu_p^2}{16 \nu_{\text{max}}^4}, \frac{1}{2}, \frac{1}{2}; 1; -\frac{\nu_p^2}{\nu_{\text{max}}^2} \right)$$

where $v_0$ is the maximum Doppler frequency caused by the scatterer motion at an average velocity of $v_0$ on its received signal, i.e., $v_0 = \frac{\nu_0}{c} \nu_{\text{max}}$. Applying the Fourier transform on the autocorrelation function results in the following Doppler power spectrum:

$$S(v) = \frac{K_0(v)}{2\pi^2 v_0},$$

where $K_0(\cdot)$ is the zeroth order modified Bessel function of the second kind.

3.3.3 Gaussian distribution: Since the velocity (magnitude) is always positive or equal to zero, we use a half-Gaussian distribution for the calculation of the autocorrelation function. In order to keep the cumulative distribution function (cdf) at $\infty$ equal to 1, a factor of $2$ is introduced in the original distribution, thus yielding the scatterer velocity pdf:

$$p(v_{\text{SG}}) = \begin{cases} \frac{2}{\sqrt{2\pi} \sigma} e^{-\frac{(v_{\text{SG}} - \mu_{v_{\text{SG}}})^2}{2\sigma^2}}, & v_{\text{SG}} \geq 0, \\ 0, & \text{otherwise}, \end{cases}$$

with $\mu_{v_{\text{SG}}} = \sqrt{\frac{\nu}{c}} \sigma$, $\sigma_{v_{\text{SG}}} = \sqrt{\frac{\nu^2 - \nu}{c^2}} \sigma$ and $\sigma^2$ is the variance parameter of the original Gaussian distribution. The autocorrelation function is computed as:

$$R(\tau) = \frac{2}{\sqrt{2\pi} \sigma} \int_0^{\infty} \int_0^{\infty} \frac{2\pi f_c}{v_p} e^{-\frac{v_{\text{SG}}}{\sigma}} dv_{\text{SG}} d\gamma_{\text{SG}} = 2F_2 \left( \frac{1}{2}, \frac{1}{2}, 1, 1; -\frac{8\pi^2 f_c^2}{v_p^2} \sigma^2 \gamma^2 \right)$$

with $\lambda_c$ being the carrier wavelength. The corresponding Doppler power spectrum is:

$$S(v) = \sqrt{2} \lambda_c G_{33}^0 \left( \frac{\lambda_c v}{\sqrt{2}} \sigma, \frac{1}{2}, \frac{1}{2}, 1, 0, 0, 0 \right).$$

3.3.4 Triangular distribution: Here, the scatterer velocity distribution is given by:

$$p(v_{\text{SG}}) = \begin{cases} \frac{2}{v_{\text{max}}^2} v_{\text{SG}}, & 0 \leq v_{\text{SG}} \leq v_{\text{max}}, \\ 0, & \text{otherwise}, \end{cases}$$

where $\mu_{v_{\text{SG}}} = \frac{v_{\text{max}}}{2}$ and $\sigma_{v_{\text{SG}}} = \frac{v_{\text{max}}}{2\sqrt{3}}$. The corresponding power spectrum is:

$$S(v) = \frac{1}{\pi} \sqrt{\frac{2\pi^2 v_{\text{max}}^2}{2\pi^2 v_{\text{max}}^2}}.$$
whith $\mu_{Sx} = \frac{v_{max}}{2}$ and $\sigma_{Sx} = \frac{v_{max}}{3 \sqrt{2}}$. In this case, the autocorrelation function is given by:

$$R(\tau) = \int_{0}^{v_{max}} J_{0}^{2} \left( \frac{2 \pi f_{c} \nu Sx}{v_{p}} \right) \left( \frac{2}{\nu_{max}^{2}} - \frac{2 \nu Sx}{v_{max}^{2}} \right) dv_{Sx}$$

$$= 2 F_{3} \left( \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 3 \right) - \frac{4 \pi^{2} f_{c}^{2} \nu_{max}^{2} \tau^{4}}{v_{p}^{4}}$$

$$- J_{0} \left( \frac{2 \pi f_{c} \nu_{max}}{v_{p}} \right)^{2} - J_{1} \left( \frac{2 \pi f_{c} \nu_{max}}{v_{p}} \right)^{3}$$

$$= 2 F_{3} \left( \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 3 \right) - \frac{4 \pi^{2} v_{max}^{2} \tau^{2}}{v_{p}^{4}}$$

$$- J_{0} \left( \frac{2 \pi v_{max} \tau}{v_{p}} \right)^{2} - J_{1} \left( \frac{2 \pi v_{max} \tau}{v_{p}} \right)^{3}. \quad (29)$$

Applying the Fourier transform on the autocorrelation function in (29), the Doppler power spectrum is obtained as

$$S(\nu) = \frac{G_{30}^{30} \left( \frac{\nu^{2}}{4 v_{max}^{2}} \right)}{2 \pi \nu v_{max}} \left( \frac{1}{2}, \frac{1}{2}, 1, 1, 0, 0, 0 \right)$$

$$- \frac{G_{22}^{20} \left( \frac{\nu^{2}}{4 v_{max}^{2}} \right)}{2 \pi \nu v_{max}} \left( \frac{1}{2}, \frac{1}{2}, 1, 0, 0, 0, 0 \right)$$

$$- \frac{G_{21}^{21} \left( \frac{\nu^{2}}{4 v_{max}^{2}} \right)}{2 \pi \nu v_{max}} \left( \frac{1}{2}, \frac{1}{2}, 1, 1, 0, 0, 0 \right). \quad (30)$$

Figures 2(a) and 2(b) show the autocorrelation functions and the Doppler power spectra, respectively, for the above four scatterer velocity distributions: uniform, exponential, half-Gaussian and triangular distributions. In order to make the autocorrelation functions and the Doppler power spectra under the four scatterer velocity distributions comparable, the distribution parameters are selected so that the mean scatterer velocities, $\mu_{Sx}$, are equal to 1 m/s. The maximum scatterer velocity, $v_{max}$, is set to 2 m/s for the uniform distribution and to 3 m/s for the triangular distribution. The mean scatterer velocity for the exponential distribution, $v_{0}$, is 1 m/s. The variance parameter of the half-Gaussian distribution, $\sigma^{2}$, is set to $\frac{\pi}{4}$, leading to an average scatterer velocity of 1 m/s. The carrier frequency, $f_{c}$, is selected to be 300 MHz.

In Figure 2(b), it is observed that the the Doppler power spectrum is bounded under the uniform and triangular distributions but not under the exponential and half-Gaussian distributions. In this figure, the Doppler frequency is depicted between -7 and 7 Hz. If the Doppler frequency range is increased, then the Doppler power spectrum continues to expand under the exponential and Gaussian distributions whereas they do not change under the uniform and rectangular distributions. This is because the scatterer velocity is bounded by the maximum velocity, $v_{max}$, under the uniform and triangular distributions while it is not bounded under the exponential and half-Gaussian distributions.

4 Model Validation

The proposed model is validated through curve-fitting and statistical testing of the empirical Doppler power spectrum curves reproduced from [6–8], and our analytical curves. The analytical curves are also compared with those from [10] in order to show the improvement of the double Doppler model comparing with the single Doppler model.

In Figure 3, the empirical curve-fitting obtained with our model are compared to those presented in [10]. The empirical curves, measured at a carrier frequency of 2.5 GHz, were extracted and reproduced from [7, Figure 7]. The difference between the two models is the introduction in our model of the double Doppler components, caused by scatterer motion with respect to the transmitter and the receiver to the reflected signals.
Figure 3. Performance comparison with the results presented in [10, Figures 10 and 11]: curve-fitting to the empirical Doppler power spectrum at (a) moderate and (b) high Doppler spread. Figure 3(a) shows a Doppler power spectrum measured in a channel with moderate Doppler spread, while Figure 3(b) shows a Doppler power spectrum measured in an environment with high Doppler spread. The analytical curves are produced using an exponential scatterer velocity distribution. This distribution is selected because it provides the best curve fitting, among the four scatterer velocity distributions aforementioned, with the empirical curves. Our model tends to spread widely and thus better follow the measurement curves, whereas the model of Roy tends to descend more steeply and to deviate from the measurement curves. The Chi-square test results show that our model outperforms the model of Roy in both moderate and high Doppler spread channels.

Figure 4 compares the proposed analytical model curves with the empirical curves reproduced from [6, Figure 4]. Figure 4(a) shows the Doppler power spectra generated by foliage movements, while Figure 4(b) represents the Doppler power spectra caused by passing vehicles, both in a fixed wireless channel at a carrier frequency of 29.5 GHz. The analytical curves for the model presented in [10] and our model are plotted under an exponential scatterer velocity distribution. It shows that our model outperforms the other model in both cases. However, while they follow relatively closely the Doppler power spectrum caused by foliage motion, neither of the two models follow the high Doppler shifts in the case of passing vehicles. Intuitively, the foliage fluctuates randomly in every direction due to the wind. Its velocity is slow and its average is close to zero. The exponential distribution also emphasizes the low speeds and therefore better describes the case of foliage motion than the case of vehicles moving at high speeds. It is conjectured that a Laplacian distribution would better describe the vehicle velocity distribution. Unfortunately, the calculation of the autocorrelation function and the Doppler power spectrum under the Laplacian distribution is currently
difficult to handle, and therefore, no analytical results for the Laplacian scatterer velocity distribution is presented at this point.

Figure 5 shows the curve fitting results for the empirical curves presented in [8, Figure 2]. The measurements were conducted for a short-range transmission at 5.3 GHz. Three exponential distributions of the scatterer velocity with different variances are used in the tests. Once again, the Chi-square statistical test shows that the Doppler power spectrum under exponential scatterer velocity distribution follows closely the measurement curve and describes well the slow time-varying nature of the short-range channel at 5.3 GHz.

5 Conclusions

In this paper, the double Doppler effects caused by scatterer motion with respect to the transmitter and the receiver are studied under different scatterer velocity distributions, including uniform, exponential, half-Gaussian and triangular distributions. Analytical curves of the Doppler power spectrum are compared and statistically tested with measurement results extracted and reproduced from empirical results published in the literature. The curve-fitting and statistical test results show that the exponential scatterer velocity distribution outperforms the other distributions and describes accurately the slow time-varying nature of the fixed wireless channels. It also shows that the proposed double Doppler model characterizes better the Doppler effects caused by scatterer mobility at low velocities than the single Doppler model does. In addition, more investigation is needed to find a suitable distribution that closely describes the scatterer velocities in fast-varying fixed wireless channels, e.g., caused by passing vehicles.

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References

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